

Gouy shift and temporal reshaping of focused single-cycle electromagnetic pulses

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We discuss exact solutions of Maxwell's equations that describe the evolution of single-cycle electromagnetic pulses. The solutions are applied to recent observations of the diffraction transformation of terahertz pulses. In particular, we elucidate the role of the Gouy shift in the temporal reshaping and polarity reversals of single-cycle terahertz pulses. © 1998 Optical Society of America

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Advances in ultrafast optical technology have made possible the generation of single-cycle and half-cycle electromagnetic pulses of subpicosecond temporal duration.¹ These pulses have center frequencies in the terahertz range and spectra that extend from zero to several terahertz. This extraordinarily large bandwidth results in significant temporal reshaping of focused terahertz pulses even when they propagate through free space or pass through apertures.^{2,3} Analysis of this diffraction-induced pulse shaping is usually carried out through numerical solution of Maxwell's equations.³ On another front there is much interest in exact solutions of Maxwell's equations that describe the localized transmission of electromagnetic energy.⁴⁻⁶ One particularly interesting class of solutions termed electromagnetic directed-energy pulse trains by Ziolkowski^{5,6} has finite total energy and could prove useful in describing ultrashort-pulse phenomena in regimes beyond the slowly varying envelope and the paraxial approximations.

In this Letter we show that a certain subset of Ziolkowski's electromagnetic directed-energy solutions of Maxwell's equations can describe most of the observed features of focused single-cycle electromagnetic pulses. These exact solutions demonstrate how the Gouy phase shift of focused beams leads to temporal reshaping and polarity reversals as terahertz pulses propagate through free space.

The modified power spectrum pulse discovered by Ziolkowski^{5,6} is an exact solution of the free-space wave equation:

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) f(\mathbf{r}, t) = 0. \quad (1)$$

In terms of the variables $\rho^2 = x^2 + y^2$, $\tau = z - ct$, $\sigma = z + ct$, and $s = \rho^2/(q_1 + i\tau) - i\sigma$, the modified power spectrum solution is given by

$$f(\mathbf{r}, t) = f_0/(q_1 + i\tau)(s + q_2) \quad (2)$$

for a pulse propagating along the z direction. Here q_1 and q_2 are parameters with dimensions of length that can be shown to determine the peak wavelength and the Rayleigh range of the pulses, respectively. For the typical terahertz pulse q_1 is a few orders of magnitude smaller than q_2 . Vector field solutions of Maxwell's equations are then readily obtained by use of the method of Hertz potentials.⁶ In this Letter we construct a Hertz vector that is transverse to the direction of propagation $\Pi = \hat{x}f(\mathbf{r}, t)$, where \hat{x} is a unit vector. The electric and the magnetic fields are then found from $\mathbf{E} = -\mu_0 \nabla \times \partial \Pi / \partial t$ and $\mathbf{H} = \nabla \times \Pi$, respectively. In Ref. 7, the use of a z -directed Hertz potential was reported to result in toroidal wave packets, termed focused doughnuts. We find here that the transversely oriented Hertz potential results in oblate wave packets that resemble focused pancakes.

For the case in which $q_1 \ll q_2$ (which means that the effective wavelength of the pulse is much shorter than the Rayleigh range), it can be shown that the dominant field components are

$$E_y(\mathbf{r}, t) = 2f_0 \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{(q_1 + i\tau)^2 - (q_2 - i\sigma)^2}{[\rho^2 + (q_1 + i\tau)(q_2 - i\sigma)]^3}, \quad (3)$$

$$H_x(\mathbf{r}, t) = 2f_0 \frac{2\rho^2 \cos(2\varphi) + (q_1 + i\tau)^2 + (q_2 - i\sigma)^2}{[\rho^2 + (q_1 + i\tau)(q_2 - i\sigma)]^3}, \quad (4)$$

where $\varphi = \tan^{-1}(y/x)$. The E_x component is exactly zero owing to the x orientation of the Hertz vector. The other components are such that E_z/E_y and H_z/H_x are of the order of $O(\sqrt{q_1/q_2})$, whereas H_y/H_x is of the order of $O(q_1/q_2)$.

Equations (3) and (4) are exact, finite-energy solutions of Maxwell's equations that describe the spatiotemporal evolution of focused single-cycle electromagnetic pulses with diffraction effects included. Owing to the linearity of Maxwell's equations, the real and the imaginary parts of Eqs. (3) and (4) separately

constitute valid pulse solutions. Furthermore, since the complex field is an analytic signal, the real (E_y^r) and imaginary (E_y^i) parts are related by a Hilbert transform at any given position \mathbf{r} .

To visualize these pulses and relate them to terahertz experiments, we set the parameters $q_1 = 0.025$ mm and $q_2 = 1000$ mm and plot in Fig. 1 the on-axis ($\rho = 0$) temporal shapes of E_y^r and E_y^i in the $z = 0$ plane (the focal plane). In this plane one of the solutions is symmetric, and the other is antisymmetric. These pulse shapes are similar to those observed in terahertz experiments.^{8,9} The symmetric pulse is often referred to as unipolar.² We also point out that since these exact solutions are nonseparable the pulse shape and width at the focus will depend on radial position ρ . This dependence is also shown in Fig. 1. The radial extent of the pulse is given roughly by $\sqrt{q_1 q_2}$ since, as can be seen from Eq. (3), when $\rho = \sqrt{q_1 q_2}$ the electric field is 1/8 of its value on axis. The temporal pulse width is also roughly $\tau_p \approx 2\sqrt{3} q_1/c$, which, for $q_1 = 0.025$ mm, is ~ 289 fs.

The spatiotemporal evolution of the imaginary pulse $E_y^i(\mathbf{r}, t)$ from a distant plane ($z = -3$ m) before the focus, through the focus, and then to a plane in the far field ($z = 3$ m) is shown in Fig. 2. The variable $z - ct$ represents the local distance measured from the pulse center. One can clearly observe the curved phase fronts of the pulse as it converges to a minimum spot size at the focus and then diverges again. More significantly, polarity reversal and substantial temporal reshaping can be observed as the pulse evolves through the focus. The real solution $E_y^r(\mathbf{r}, t)$ also undergoes a similar transformation that is due simply to the well-known Gouy shift of focused beams.¹⁰ In fact, experiments with one-cycle pulses should provide a simple and direct way to observe the Gouy effect.

The diffraction-induced transformation of pulse shapes can be seen more clearly in Fig. 3, in which the on-axis ($\rho = 0$) temporal profiles at several propagation distances are plotted. It can be seen that the symmetric real solution at $z = 0$ evolves in the far field into an inverted version of the antisymmetric imaginary solution at $z = 0$. Simultaneously, the antisymmetric imaginary solution at $z = 0$ evolves in the far field into the symmetric real solution. These transformations can also be understood in terms of the Gouy phase shift. For short pulses, such that $c\tau_p \ll q_2$, the first term in the numerator of Eq. (3) can be neglected to yield the real and the imaginary parts of the on-axis field:

$$E_y^r(\rho = 0, z, T) = \frac{A(T)}{(q_2^2 + 4z^2)^{1/2}} \cos[\alpha(T) + \phi(z)],$$

$$E_y^i(\rho = 0, z, T) = \frac{A(T)}{(q_2^2 + 4z^2)^{1/2}} \sin[\alpha(T) + \phi(z)],$$

(5)

where $A(T) = -2f_0\mu_0c/[q_1^3(T^2 + 1)^{3/2}]$, $\alpha(T) = 3 \times 10^{-1}(T)$, $\phi(z) = \tan^{-1}(2z/q_2)$ is the Gouy phase shift, and $T = -\tau/q_1$ is a normalized local time. Equations (5) show that the evolution of the temporal profiles during propagation is completely determined by

the Gouy phase shift $\phi(z)$, since the functional forms of $A(T)$ and $\alpha(T)$ are invariant on propagation. The scale factor $(q_2^2 + 4z^2)^{-1/2}$ simply accounts for energy conservation. In propagating from the focus to the far field $\phi(z)$ goes from zero to $\pi/2$, thus effecting the transformations $E_y^r \rightarrow -E_y^i$ and $E_y^i \rightarrow E_y^r$.

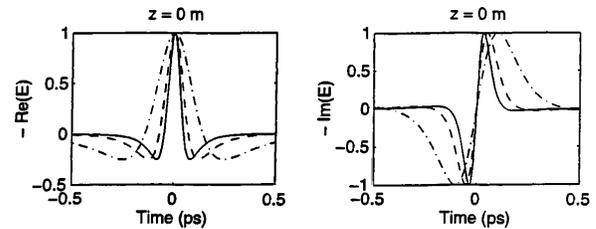


Fig. 1. Temporal profiles $E_y^r(0, \rho, t)$ (left) and $E_y^i(0, \rho, t)$ (right) at $\rho = 0$ (solid curves), $\rho = w_e$ (dashed curves), and $\rho = 2w_e$ (dotted-dashed curves) at the focal plane ($z = 0$). Here $w_e = (q_1 q_2/2)^{1/2}$ is an effective beam radius at the focus. The pulse width increases off axis. Here and in Figs. 2-4 $q_1 = 0.025$ mm and $q_2 = 1000$ mm.

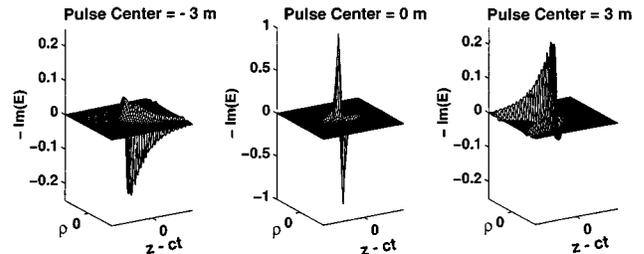


Fig. 2. Spatiotemporal evolution of the imaginary solution $E_y^i(\mathbf{r}, t)$ from $z = -3$ m before the focus (left), through the focus (center), and then to the far field $z = 3$ m (right). Note the polarity reversal and temporal reshaping.

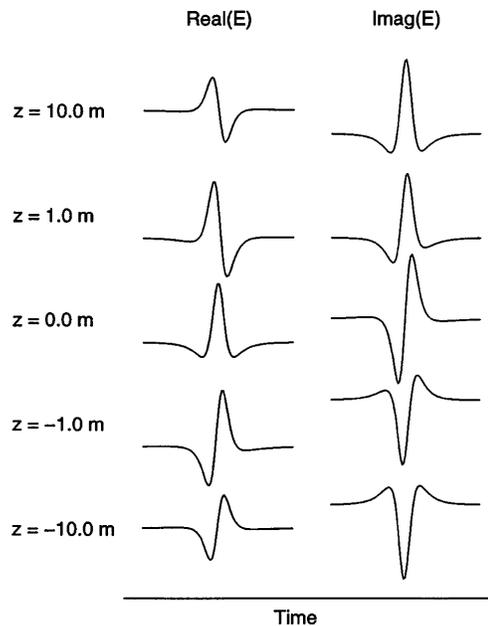


Fig. 3. Spatial evolution of the pulse shape on the axis ($\rho = 0$) at five propagation distances Z . Clearly, the real and the imaginary pulses evolve into each other during propagation.

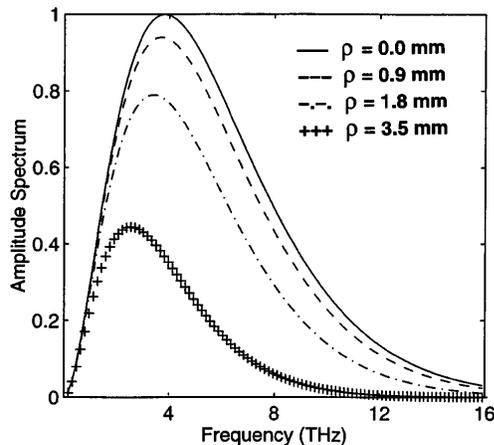


Fig. 4. Amplitude spectra at $z = 0$ for various radial positions. Note the red shift as one moves from the center of the pulse to the edge. The effective beam radius is $w_e = 3.5$ mm. The spectra are normalized by the peak value at $\rho = 0$.

Similarly, in passing from $-\infty$ to $+\infty$ through the focus, the field acquires a $\phi(z) = \pi$ phase shift that causes the inversions $E_y^r \rightarrow -E_y^r$ and $E_y^i \rightarrow -E_y^i$. The transformations described above are also obtainable from the Hilbert transform relationship between the real and the imaginary parts of the complex field. We note that in experiments involving diffraction of a terahertz pulse by a slit a similar transformation from a symmetric to an antisymmetric temporal profile was observed.^{2,3}

We obtain the approximate amplitude spectra by taking the Fourier (time) transform of Eq. (3), neglecting the first term in its numerator. From the real part we obtain

$$\begin{aligned} \tilde{E}_y^r(\mathbf{r}, \omega) = & \frac{-f_0 \mu_0 \pi}{\sqrt{q_2}} \left(\frac{|\omega|}{c} \right)^{3/2} \exp\left(-\frac{|\omega|}{c} q_1\right) \\ & \times \frac{\exp(-ikz + i\frac{\omega}{|\omega|} \phi)}{w} \exp\left(-ik \frac{\rho^2}{2R} - \frac{\rho^2}{w^2}\right), \end{aligned} \quad (6)$$

where $w^2 = w_0^2[1 + (2z/q_2)^2]$, $R = z[1 + (q_2/2z)^2]$, $k = \omega/c$, and $w_0^2 = \lambda q_2/2\pi$. Each frequency component of the pulse diffracts as a monochromatic Gaussian beam with radius $w(z)$, minimum radius (beam waist) w_0 , radius of curvature $R(z)$, and Gouy phase shift $\phi(z)$. The parameter q_2 thus plays the role of the confocal parameter, which is twice the Rayleigh range. Note that for this pulse all the frequency components are characterized by the same value of q_2 and hence will all have the same Rayleigh range. From the definition of the beam waist w_0 it is clear that if q_2 is fixed the longer wavelength components will have a larger spot size at the focus. This frequency dependence of the spot size is indeed observed in experiments

with focused terahertz beams.⁹ Figure 4 shows the amplitude spectrum at the focus for different radial positions. Note the red shift of the wavelength λ_p at the peak of the spectrum as one moves off axis. The bell-shaped spectra are also similar to those observed in terahertz experiments.^{8,9} The amplitude spectrum $|\tilde{E}_y^r(\mathbf{r}, \omega)|$ on axis is maximum at the frequency $\omega_p = 2c/q_1$; hence q_1 is related to the peak wavelength of the pulse by $q_1 = \lambda_p/\pi$. In the far field the amplitude spectra, and hence the pulse width, become independent of radial position.

We note that a recent paper by You and Bucksbaum¹¹ also discusses pulse reshaping and polarity reversal of a unipolar pulse. There the authors use an approach based on propagating the individual Fourier components of a pulse that is Gaussian in space and time. The exact solutions presented here¹² are in agreement with the results of Ref. 11.

In conclusion, we have examined the physical properties of a two-parameter exact solution of Maxwell's equations that is capable of describing focused single-cycle electromagnetic pulses. The solution clearly shows the role of the Gouy phase shift in the temporal reshaping and polarity reversals of terahertz pulses.

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