Direct Observation of the Gouy Phase Shift with Single-Cycle Terahertz Pulses

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We report a direct observation of the predicted polarity reversal due to the Gouy phase shift for single-cycle terahertz pulses passing through a focus.

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In 1890 Gouy showed that a focused electromagnetic beam will acquire an additional axial 180° phase shift with respect to a plane wave as it evolves through its focus [1–3]. This phase shift has important consequences throughout the electromagnetic spectrum. It explains the phase advance postulated by Fresnel for the secondary Huygens' wavelets emanating from a primary wave front [4]. In curved mirror laser cavities the Gouy phase shift is responsible for the difference in resonance frequencies of different transverse modes [3]. In nonlinear optics the Gouy shift can drastically reduce the efficiency of the generation of odd order harmonics with focused beams [5]. It also plays a role in the lateral trapping force at the focus of optical tweezers [6] and leads to phase velocities that exceed that of a plane light wave in vacuum [3,7].

Although a hundred years have passed since Gouy made his discovery, efforts are still being made to provide a satisfying physical interpretation of this phase jump and to place it within the context of other phase anomalies in modern physics [8–11]. A recent analysis has interpreted the Gouy shift as a geometrical quantum effect arising, via the uncertainty principle, from a modification through focusing of the volume of space available for the propagation of the photon [9]. While satisfying in its simplicity, the appeal to quantum mechanics appears unnecessary since Gouy showed that the phase jump exists for any and all waves, sound waves included, that pass through a focus [2]. This is reminiscent of the debate as to whether the Berry phase in the polarization of light waves is a purely classical phenomenon or one that is inherently quantum mechanical [12,13]. The connection to the Berry phase is actually more than a passing one. Berry's phase is an additional geometric (topological) phase acquired by a system after a cyclic adiabatic evolution in parameter space [14]. In recent papers it has been shown that the Gouy phase jump is another manifestation of a general Berry phase where the parameter that is cycled is the complex wave front radius of curvature q associated with a Gaussian beam [10,11]. A common feature of all the modern interpretations of the Gouy shift is the notion of anholonomy; that is, the phase depends purely on the geometry of the parameter space and the circuit traversed.

The published experimental observations of the Gouy shift have been based on interferometric measurements. Gouy used white light in his original experiment and observed the axial changes in the interference pattern between a plane wave and a spherical wave [1,2]. Carpenter conducted microwave experiments in which he measured the nodes of a standing wave and inferred the existence of an additional half wave shift at the focus [15]. Recently it has been pointed out that the Gouy shift should result in polarity reversals of single-cycle terahertz pulses as they evolve through a focus [16,17]. Because terahertz measurements are sensitive to both amplitude and phase, this should enable direct, noninterferometric observations of the Gouy shift.

In this paper we present a direct and unambiguous observation of the polarity reversal due to the Gouy phase shift of focused single-cycle terahertz pulses. To our knowledge, this is the first noninterferometric observation of the Gouy shift and permits characterization of the absolute phase of a pulse. By comparing focused and collimated pulses we find that the Gouy shift is indeed a geometrical phase that exists over and above any dynamical phase acquired on propagation.

The experimental arrangement for observing the Gouy shift is shown schematically in Fig. 1. Single-cycle THz pulses are generated and detected photoconductively in a manner similar to that of Ref. [18]. The system is aligned linearly using large-aperture, transmissive, high resistivity, silicon optics (50.4 mm diameter) and aplanatic Si lenses (10 mm diameter) to couple the THz radiation into or out of the photoconductive antennas. 100-fs Ti:sapphire laser pulses at 800 nm are used to drive the photoconductive transmitter (TX) and detector (RX) with 30 mW of average optical power. Our antenna design uses a dipolar geometry with a 5-μm gap located in the center of coplanar transmission lines, and the antennas are fabricated on low-temperature-grown epitaxial layers of GaAs (LT-GaAs). The THz electric field was generated by biasing the transmitting antenna with 30 V as the optical pulses were focused into its photoconductive gap. This output from the terahertz source was collimated by the Si lens L1 ($f = 127$ mm) and focused onto the detector by an identical lens, L2. The THz pulse was sampled by varying the time delay between the generated electric field and the optical gating/probe pulse at the detector. Modulating the optical pump beam at 5 kHz made it possible to measure

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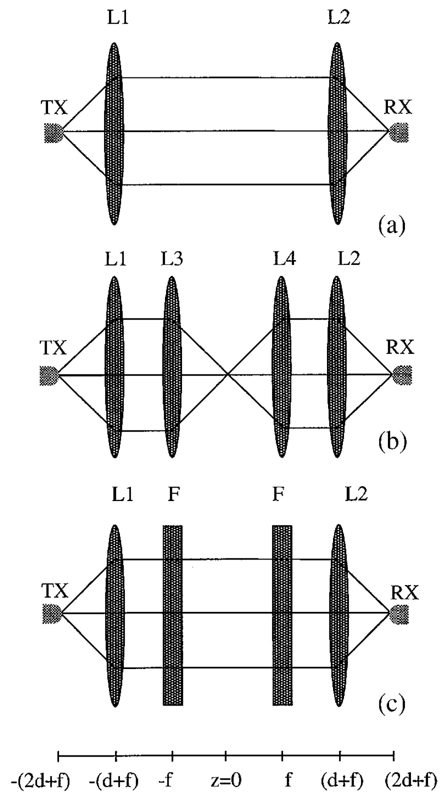

FIG. 1. Schematic of experimental setup showing the lens configurations: (a) collimated configuration, (b) focused configuration, and (c) collimated configuration, where L3 and L4 of (b) are replaced by silicon flats, F. Here TX and RX represent the photoconductive transmitter and receiver, respectively.

the photocurrent generated by the incident electric field and the gating optical pulse with the use of a current preamplifier and a lock-in amplifier. The experiment consisted of measuring the transmitted pulse at the receiver for the different configurations shown in Fig. 1.

The results of the experiment are shown in Fig. 2. The pulse shown as the dashed curve is the measured terahertz signal at the receiver in the collimated configuration of Fig. 1(a). The pulse width (FWHM) of the main signal is approximately 1 ps and the polarity is positive. Note that the measurement represents the photocurrent excited by the probe laser pulse at the detector and controlled by the electric field of the terahertz pulse. Because of the finite response time of the detector (~800 fs) the measured signal is actually a convolution of the detector response with the terahertz pulse shape. The direction of the photocurrent is determined by the polarity of the time-varying electric field of the terahertz pulse, which acts as a bias.

The solid curve in Fig. 2 is the measured terahertz signal in the focused configuration of Fig. 1(b). Clearly the polarity of this "focused" pulse is reversed with respect to that of the "collimated" pulse. This is in agreement with the predictions of Refs. [16,17] and is a direct consequence of the Gouy phase shift. In addition, the focused pulse appears slightly narrower than the collimated

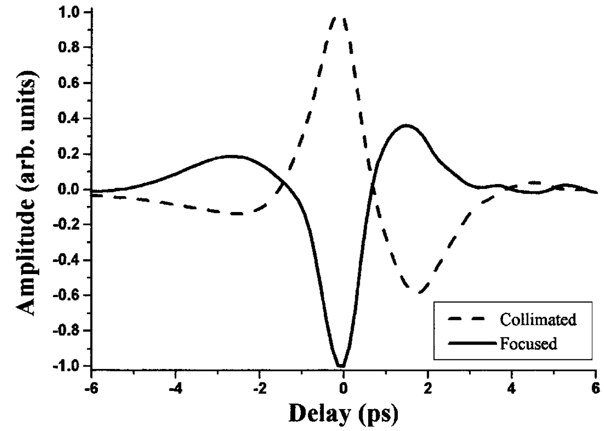

FIG. 2. Dashed curve: measured terahertz pulse in the collimated configuration of Fig. 1(a). Solid curve: measured terahertz pulse in the focused configuration of Fig. 1(b) showing polarity reversal.

pulse. This is probably due to the improved focusing of the high frequency components and greater loss of the low frequency components in the focused configuration. We note that the pulses in Fig. 2 are shown overlapped in time for ease of comparison. In reality there is a group delay of 32.4 ps between the focused and collimated pulses owing to the finite thickness of the extra silicon lenses in the confocal geometry.

To confirm that the observed polarity reversal is due to the passage through a focus and not due to a linear phase shift on propagation or to dispersive effects in the extra two lenses, these lenses were replaced by a pair of high-resistivity, silicon flats of comparable optical thickness. The result of this control experiment is shown in Fig. 3. The dashed curve is the same reference collimated pulse of Fig. 1(a) while the solid curve shows the collimated pulse with the two silicon flats of Fig. 1(c). There is no polarity reversal in this case and the two pulses are almost

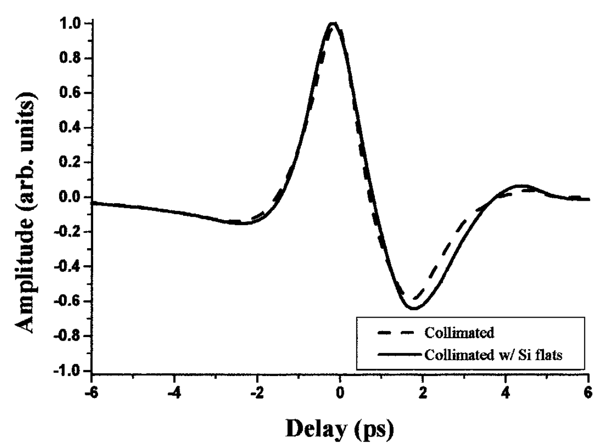

FIG. 3. Dashed curve: collimated terahertz pulse as in Fig. 1(a). Solid curve: terahertz pulse in the system with lenses L3 and L4 replaced by silicon flats [Fig. 1(c)]. There is no polarity reversal in this case.

identical. Here, too, the pulses have been overlapped in time for clarity although the two silicon flats introduce a similar pulse delay.

The experiment described here provides not only a direct and dramatic observation of the Gouy effect but also yields a quantitative measurement of the phase shift. The phase shift is extracted from the complex Fourier transforms of the collimated and focused temporal pulse profiles. Numerically computed amplitude spectra for the two pulses are shown in Fig. 4. The spectra peak around 0.2 THz and extend beyond 1 THz. It is seen that the spectrum of the focused pulse (solid curve) is slightly blueshifted with respect to that of the collimated pulse (dashed curve) and is richer in the higher frequencies. This is a result of the frequency-dependent Rayleigh range for beams emerging from a fixed aperture. This leads to improved focusing of the high frequency components and explains the slight pulse narrowing observed in Fig. 2. The spectra have a low frequency cutoff at about 30 GHz due to the finite aperture of the lenses.

Figure 5 (solid curve) shows the relative phase of the frequency components of the focused pulse with respect to the collimated pulse. The phase shift starts at zero for dc and rapidly increases to 180° at higher frequencies. In extracting the Gouy phase shift, the pulses were first numerically overlapped in time before the Fourier transforms were computed. This removes the dynamical phase e^{ikz} due to linear propagation in the finite thickness of the two extra lenses L3 and L4. Here it is also assumed that the high-resistivity ($\rho > 10$ kΩ cm) Si lenses used have negligible dispersion. This is indeed a valid assumption over the frequency range of our terahertz pulses as demonstrated by Grischkowsky *et al.* [19].

The dashed curve in Fig. 5 is a theoretical prediction of the geometrical phase difference between the focused and collimated configurations based on a simple Gaussian beam analysis. As shown in Ref. [20], the excitation geometry used here produces terahertz pulses that have a near-Gaussian transverse profile. Propagation through a system of dispersionless lenses can be analyzed in the frequency domain by applying the standard *ABCD* matrices of paraxial wave optics to each frequency component of the incident pulse [3]. The collimated and focused configurations in Fig. 1 differ only by the presence of the two extra lenses L3 and L4 in Fig. 1(b). It is thus sufficient to calculate the phase shift imparted by those two lenses (of focal length f) spaced by $2f$ compared to free-space propagation over the same distance. The *ABCD* matrix for propagation from a plane immediately before L3 to a plane immediately after L4 is given by

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & 2f \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix},$$

where the first and last matrices represent the effect of the lenses while the second matrix describes the free-space propagation over a distance of $2f$. Upon carrying out the matrix multiplication we find $A = -1$ and $B = 2f$ for the sequential transformation by lenses L3 and L4 (focused configuration) compared to $A = 1$ and $B = 2f$ for the free-space transport over the distance $2f$ without the two lenses (collimated configuration). The Gouy phase ψ for this transformation is given in terms of the complex beam parameter q_1 by [3]

$$\exp(i\psi) = \frac{A + B/q_1}{|A + B/q_1|}, \tag{1}$$

where

$$\frac{1}{q_1} = \frac{1}{R_1} - i\,\frac{\lambda}{\pi w_1^2},$$

R_1 is the radius of curvature, w_1 is the beam radius, and λ is the wavelength of the Fourier component of the pulse incident on L3. Assuming plane phase fronts incident on L3 ($R_1 \to \infty$, $w_1 \equiv w_0$) the Gouy phase is

$$\psi = \tan^{-1}\left[\frac{B}{A}\,\frac{\lambda}{\pi w_0^2}\right]. \tag{2}$$

Because of the multivalued nature of the arctangent function, care must be taken to place ψ in its proper quadrant by examining $\cos\psi$ and $\sin\psi$ in Eq. (1). The Gouy phase

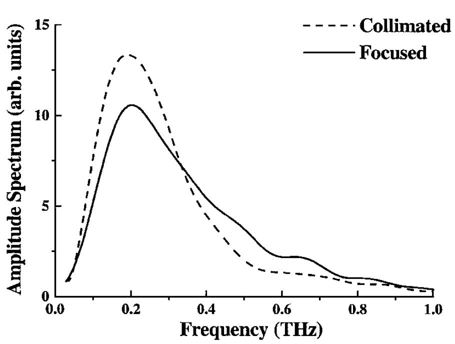

FIG. 4. Amplitude spectra of the collimated and focused (solid curve) pulses.

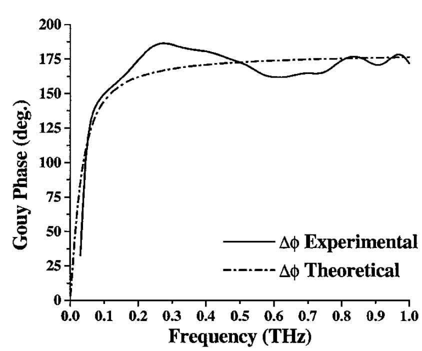

FIG. 5. Measured Gouy phase shift (solid curve) versus frequency. The dashed curve is a theoretical fit based on Eq. (3).

difference between the focused and collimated geometries is thus found to be

$$\Delta\psi = \pi - 2\tan^{-1}\left(\frac{2}{\pi}\frac{\nu_c}{\nu}\right), \tag{3}$$

where ν is the frequency in Hz, $\nu_c = \frac{fc}{w_0^2}$, and c is the speed of light. The theoretical curve in Fig. 5 is a plot of this function with $\nu_c = 0.05$ THz. Clearly $\Delta\psi$ reaches an asymptotic value of 180° for high frequencies and grows linearly with slope of π/ν_c for small values of ν. The frequency ν_c thus represents a "corner frequency" where the π asymptote intersects the line π/ν_c. Physically, when $\nu = \nu_c$, the Rayleigh range of the collimated pulse at that frequency is $z_R = \pi f$. For the focused pulse the Rayleigh range at ν_c is approximately f/π, which allows the Gouy phase difference to build up close to π over the propagation distance of $2f$. This corner frequency is easily read off the experimental plot of Gouy phase versus frequency. We use ν_c instead of the initial beam radius w_0 as the key experimental parameter in Eq. (3) because of the difficulty in determining accurately the width of a terahertz beam.

The theoretical expression for $\Delta\psi$ in Eq. (3) explains the key experimental features such as the asymptotic value of π for the phase shift, the presence of a sharp break at a corner frequency, and the linear dependence at low frequencies. The phase oscillations in the experimental data are due to diffraction effects from the edges of the focusing lenses [7]. At 50 GHz (the corner frequency) the beam radius is 27 mm which is about the size of the focusing optics. A detailed fit of the experimental data to theory would require a numerical integration of the broadband Huygens-Fresnel diffraction integral [7].

In conclusion, we have observed for the first time the polarity reversal due to the Gouy phase shift for single-cycle pulses evolving through a focus. This has made it possible to observe and measure the Gouy phase shift in a direct, noninterferometric manner and confirm the interpretation of the Gouy phase as a geometrical effect.

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