Polarization instabilities in birefringent nonlinear media: application to fiber-optic devices

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The intensity-dependent refractive index leads to an instability in the polarization state of an intense light beam oriented along the fast axis of a birefringent nonlinear medium. Depending on initial conditions, the spatial evolution of the polarization state can be oscillatory or rotatory, in a manner analogous to the motion of a nonlinear pendulum.

A medium with linear birefringence will preserve the polarization state of a light beam whose electric-field vector is oriented along either principal axis of the medium. In that regard, the two principal axes are entirely equivalent. It is usually assumed that this equivalence also holds at high intensities and that an intense beam oriented along either axis will suffer no changes in its polarization, barring imperfections that cause scattering between the axes. In this Letter, we show that the intensity-dependent refractive index leads to an instability in the polarization state of a light wave oriented along the fast axis of a birefringent medium. The slow axis remains a stable guiding center. Depending on the input intensity and the polarization state, the polarization ellipse can execute either oscillatory or rotatory motions about the slow axis in a manner analogous to the motion of a nonlinear pendulum. The results presented here are important both for fiber-optic devices that rely on an intensity-dependent polarization state in a birefringent fiber[1–3] and for those nonlinear devices in which the preservation of linear polarization is essential.

The spatial evolution of the polarization state in a birefringent nonlinear medium is conveniently described by coupled wave equations for the orthogonal circularly polarized modes c_+ and c_- (Ref. 4):

$$dc_+/dz = i\kappa c_- + i\beta|c_-|^2 c_+, \tag{1a}$$

$$dc_-/dz = i\kappa c_+ + i\beta|c_+|^2 c_-. \tag{1b}$$

The circular-mode amplitudes are coupled because of the linear birefringence δn through $\kappa = \pi\delta n/\lambda$ and thus periodically exchange energy as they propagate. The cross-phase modulation terms in β lead to an intensity-dependent phase difference between the two modes and hence to a rotation of the polarization ellipse.[5] The nonlinear coefficient β is proportional to the self-focusing index n_2. For an optical fiber of effective area A_{eff} and refractive index n, we can write $\beta = 4\pi\chi/3\lambda$ (W cm)$^{-1}$, where $\chi = 4\pi n_2 \times 10^7/ncA_{\mathrm{eff}}$. Scaling of the variables in Eqs. (1) yields a critical power $P_c = 2\kappa/\beta$ W.

For weak fields we may neglect the nonlinear terms in Eqs. (1). The coupled equations then reduce to

$$d^2c_\pm/dz^2 + \kappa^2 c_\pm = 0, \tag{2}$$

which represents a simple harmonic oscillator with complex displacement. The polarization state is determined by the complex ratio $\xi = c_+/c_-$. The azimuth of the polarization ellipse is $\theta = \frac{1}{2}\arg(\xi)$, and the ellipticity is given by $e = (|\xi| - 1)/(|\xi| + 1)$. From Eq. (2) it is easy to see that the ellipticity and the azimuth of the polarization ellipse are oscillatory functions of distance along the propagation direction, with a period given by the beat length $L_0 = \pi/\kappa$.

The evolution of the polarization state of the light beam during propagation can be represented by a variety of graphic methods. Two particularly useful representations are the Poincaré sphere[6] and the phase plane.[7] We use the latter in order to point out the analogy to the motion of a plane pendulum. On the phase plane [Fig. 1(a)] the azimuth indicates the orientation of the polarization ellipse as measured from the slow axis. The ellipticity is analogous to the velocity of the pendulum. States of linear polarization and various azimuthal angles are represented on the line $e = 0$. In particular, the point C_1 ($\theta = 0°$, $e = 0$) represents linear polarization along the slow axis and C_2 ($\theta = 90°$, $e = 0$) represents linear polarization along the fast axis. Note that $\theta = 90°$ and $\theta = -90°$ are indistinguishable polarization states, and thus the phase plane should be considered rolled around a cylinder so that the points at $\theta = \pm 90°$ are superimposed. The lines $e = 1$ and $e = -1$ represent right and left circularly polarization states, respectively.

Starting from an arbitrary input polarization state, say $\theta = 30°$, $e = 0$, the polarization evolves with distance along a trajectory indicated by the arrows in Fig. 1(a) and all the orbits close after a distance equal to the beat length. Closed orbits represent oscillatory motions. Input orientations of less than 45° from the slow axis lead to oscillatory motion about that axis, whereas larger angles result in oscillations about the fast axis. At low intensities both axes are stable centers. A linearly polarized input beam oriented along either axis will maintain its polarization.

As the input intensity is increased, the nonlinear terms lead to intensity-dependent phase shifts between the two coupled modes and, consequently, a

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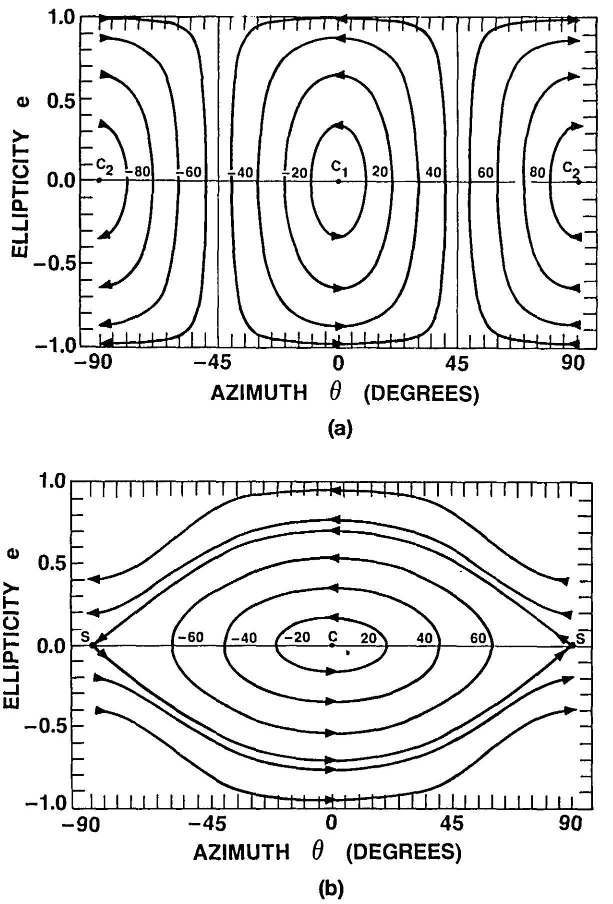

Fig. 1. Phase-plane trajectories of the polarization state. (a) Low input power ($p \ll 1$). (b) High input power ($p = 3$). The input power is normalized by the critical power P_c. Points labeled S are unstable saddle points.

nonlinear rotation of the polarization ellipse. The resulting changes in the phase plane trajectories are shown in Fig. 1(b) for a normalized input power of $p = 3$. For this input power the slow axis is still a stable center. The trajectory that passes through ±90° is a separatrix orbit with the points of 90° being unstable saddle points (corresponding to a pendulum standing on its head). The separatrix divides the phase plane into regions of oscillatory motion (closed orbits) and regions of rotatory motion. Thus, for example, an input beam launched at 60° with linear polarization will oscillate about the slow axis, while one with an ellipticity of 0.5 will execute complete rotations. Because the fast axis now corresponds to an unstable saddle point, any small deviation from perfect linearity or perfect orientation along that axis will lead to large changes in the output polarization.

As a typical application, consider an input beam linearly polarized at angle θ_0 to the slow axis of a birefringent fiber. The fiber length is chosen equal to the beat length (or an integer multiple thereof) so that at low intensities the input polarization state is reproduced at the output. A polarizer is placed at the exit and oriented at angle $\theta_0 + 90°$ so as to reject the output polarization at low intensity. When the input power is increased, self-induced polarization changes lead to nonlinear transmission through the exit polarizer. The nature of this transmission differs dramatically depending on which principal axis the input polarization angle is close to. As is shown in Fig. 2, for θ_0 near 0° (slow axis) the tranmission stays close to zero for increasing input power. The same is true for θ_0 exactly equal to 90°. However, even the slightest misalignment leads to substantial nonlinear transmission for beams oriented close to the fast axis ($\theta_0 \simeq 90°$). This is because the fast axis becomes an unstable saddle point at high power.

The origin of the instability is the intensity-dependent refractive index, which tends to reduce the linear birefringence for beams oriented along the fast axis. The resulting change in the beat length is shown in Fig. 3 (solid line) as a function of the input power normalized by $2\kappa/\beta$. At a critical input power, the beat length diverges rapidly as the induced birefringence cancels the existing linear birefringence δn. For beams oriented along the slow axis, the birefringence increases with input power and thus the beat length decreases monotonically (dashed line). Analytic expressions for the power-dependent beat length are given in Appendix A.

The critical power required for the fast-axis instability scales linearly with the birefringence. A bire-

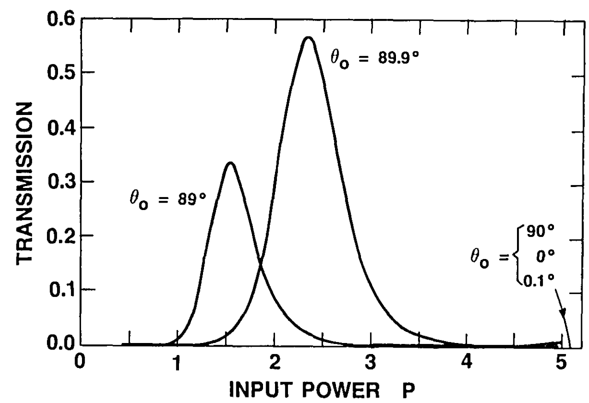

Fig. 2. Transmission of a birefringent fiber and crossed polarizer combination as a function of the normalized input power for different input angles. Here $\kappa L = \pi$.

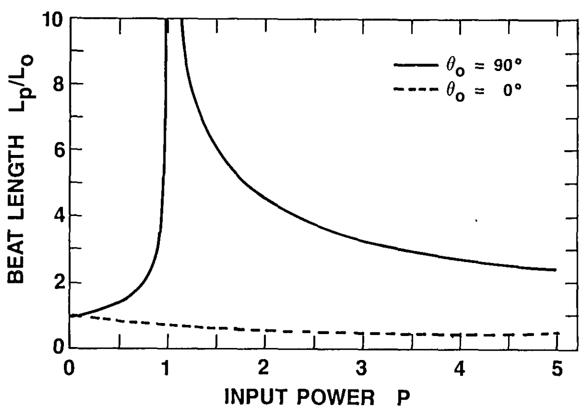

Fig. 3. Change of beat length owing to intense beam oriented along the fast axis (solid line) or along the slow axis (dashed line).

fringent fiber with $\delta n = 10^{-5}$, core diameter 4 μm, and $n_2 = 1.1 \times 10^{-13}$ esu will exhibit a fast-axis instability at peak input power levels of the order of 6 kW. For applications that require an intensity-dependent polarization state, such as optical logic and pulse shaping, relatively low-birefringence fibers should be used. In applications (such as the soliton laser[8]) in which preservation of the input polarization state at high intensities is important, care must be taken to align the beam along the slow axis of a high-birefringence fiber. In general, whenever the field-induced birefringence is of the order of the existing linear birefringence δn, an instability of the fast axis should be expected.

Appendix A

Consider a linearly polarized input beam of normalized power p and azimuth θ_0. Let $q = 1 + p \exp(i2\theta_0)$ and $u = |c_+|^2/P_c$. A solution of Eqs. (1) is[4]

$$u(z) = \tfrac{1}{2}(p - \{2[|q| - \mathrm{Re}(q)]\}^{1/2}\, cn(x|m)), \quad \text{(A1)}$$

where $cn(x|m)$ is a Jacobian elliptic function with argument

$$x = 2\kappa z |q|^{1/2} + K(m),$$

quarter-period $K(m)$, and parameter

$$m = \tfrac{1}{2}[1 - \mathrm{Re}(q)/|q|].$$

Since $cn(x|m)$ has period $4K(m)$ in x, the power-dependent beat length is

$$L_p/L_0 = 2K(m)/\pi[1 + p^2 + 2p\cos(2\theta_0)]^{1/4}. \quad \text{(A2)}$$

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