Energy storage in superluminal barrier tunneling: Origin of the "Hartman effect"

Herbert G. Winful

Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI 48109
winful@eecs.umich.edu

Abstract: We show that the anomalously short delay times observed in barrier tunneling have their origin in energy storage and its subsequent release. The observed group delay is proportional to the energy stored. This delay is not a propagation delay and should not be linked to a velocity since evanescent waves do not propagate. The "Hartman effect", in which the group delay becomes independent of thickness for opaque barriers, is shown to be a consequence of the saturation of stored energy with barrier length.

©2002 Optical Society of America

OCIS codes: (240.7040) Tunneling; (260.2160) Energy Transfer; (260.2110) Electromagnetic Theory; (350.2460) Filters, interference

---

References and links

1. L. A. MacColl, "Note on the transmission and reflection of wave packets by potential barriers," Phys. Rev. **40**, 621-626 (1932).
2. T. E. Hartman, "Tunneling of a wave packet," J. Appl. Phys. **33**, 3427-3433 (1962).
3. Th. Martin and R. Landauer, "Time delay of evanescent electromagnetic waves and analogy to particle tunneling," Phys. Rev. A **45**, 2611-2617 (1992).
4. R. Y. Chiao, P. G. Kwiat, and A. M. Steinberg, "Analogies between electron and photon tunneling: a proposed experiment to measure photon tunneling times," Physica B, **175**, 257-262 (1991).
5. R. Y. Chiao and A. M. Steinberg, "Tunneling times and superluminality," Progress in Optics **37**, 347-406 (E. Wolf, ed., Elsevier, Amsterdam, 1997).
6. A. M. Steinberg, P. G. Kwiat, and R. Y. Chiao, "Measurement of the single-photon tunneling time," Phys. Rev. Lett. **71**, 708-711 (1993).
7. Ch. Spielmann, R. Szipocs, A. Stingl, and F. Krausz, "Tunneling of optical pulses through photonic band gaps," Phys. Rev. Lett. **73**, 2308-2311 (1994).
8. S. Longhi, M. Marano, P. Laporta, and M. Belmonte, "Superluminal optical pulse propagation at $1.5\mu m$ in periodic fiber Bragg gratings," Phys. Rev. E **64**, 055602(R) 1-4 (2001).
9. A. Enders and G. Nimtz, "Evanescent-mode propagation and quantum tunneling," Phys. Rev. E **48**, 632-634 (1993).
10. Y. Japha and G. Kurizki, "Superluminal delays of coherent pulses in nondissipative media: a universal mechanism," Phys. Rev. A **53**, 586-590 (1996).
11. R. Landauer, "Light faster than light?" Nature, **365**, 692-693 (1993).
12. H. G. Winful, "The nature of 'superluminal' barrier tunneling," Phys. Rev. Lett., to be published.
13. C. G. Montgomery, R. H. Dicke, and E. M. Purcell, *Principles of Microwave Circuits* (McGraw-Hill, New York, 1948).
14. G. Kishi and K. Nakazawa, "Relations between reactive energy and group delay in lumped-constant networks," IEEE Trans. Circuit Theory, **CT-10,** 67-71 (1963).
15. H. J. Carlin, "Network theory without circuit elements," Proc. IEEE **55**, 482-497 (1967).
16. G. D'Aguanno, et al, "Group velocity, energy velocity, and superluminal propagation in finite photonic band-gap structures," Phys. Rev. E **63**, 036610, 1-3 (2001).
17. F. T. Smith, "Lifetime matrix in collision theory," Phys. Rev. **118**, 349-356 (1960).
18. C. G. B. Garrett and D. E. McCumber, "Propagation of a Gaussian light pulse through an anomalous dispersion medium," Phys. Rev. A **1**, 305-313 (1970).
19. G. Diener, "Energy transport in dispersive media and superluminal group velocities," Phys. Lett. A **235**, 118-124 (1997).
20. M. Ware, S. A. Glasgow, and J. Peatross, "Role of group velocity in tracking field energy in linear dielectrics," Opt. Express **9**, 506-518 (2001).

#1899 - $15.00 US Received November 29, 2002; Revised December 08, 2002

21. T. Emig, Propagation of an electromagnetic pulse through a waveguide with a barrier: A time domain solution within classical electrodynamics," Phys. Rev. E **54**, 5780-5787 (1996).
22. J. M. Deutch and F. E. Low, "Barrier penetration and superluminal velocity," Ann. Phys. (NY) **210**, 184-202 (1993).
23. A. D. Jackson, A. Lande, and B. Lautrup, "Apparent superluminal behavior in wave propagation," Phys. Rev. A **64**, 044101, 1-4 (2001).

The question of how long it takes a wave packet to tunnel through a potential barrier has occupied physicists for decades ever since MacColl suggested that there is "no appreciable delay" in the transmission of the packet through the barrier [1]. Hartman later showed that there is a finite delay but that this delay becomes independent of barrier thickness for sufficiently thick barriers [2]. This saturation of delay with increasing barrier thickness has been dubbed the "Hartman effect" and its origin is considered "a poorly resolved mystery" [3]. Obviously, a lack of dependence of tunneling time on barrier length would imply arbitrarily large, and indeed, superluminal velocities for tunneling wave packets, if tunneling were in fact a *propagation* phenomenon. This has always been the assumption.

In the last decade, the well known analogy between the quantum-mechanical tunneling of particles and the classical electromagnetic tunneling of evanescent waves [3,4] has been exploited in a number of elegant experiments that have measured anomalously short delay times for tunneling through photonic bandgap structures [5-8] and "undersized" waveguides. [9]. From these delay times, group velocities much larger than c have been inferred. The usual explanation for these superluminal velocities and the lack of conflict with causality is that the pulse is merely reshaped: the earlier parts of the pulse are attenuated less than the later parts, leading to a forward shift of the center of gravity of the pulse [5,6,10]. While this argument may hold for pulses that are indeed reshaped in the tunneling process, it does not explain, in any simple, physical way, why the delay time should become independent of barrier length. It is safe to say that "our understanding of this is not all that it deserves to be" [11].

A clue to the origin of these anomalously short delay times lies in the very fact that they saturate with increasing barrier length. If a wave packet takes no more time to cover distance $10d$ than it does to cover a distance d, we must either imbue the packet with the intelligence to increase its speed by just the right amount to cover the greater distance in the same time or consider the possibility that we are not dealing with a propagation phenomenon. We have taken the latter approach in recent work where we show that tunneling is a quasi-static process in which an input pulse, much longer than the barrier width, modulates the amplitude of a standing wave in the barrier [12]. Standing waves do not go anywhere: they stand and wave, with all moving parts in the same time phase. They can also store energy. Energy storage can lead to phase shifts and time delays between input and output. Here we show that the Hartman effect can be explained in a simple, physical way by the saturation of stored energy with increasing length in a barrier. Along the way we show the direct connection between stored energy and group delay ("phase time"), a connection well known in microwave circuits and lumped element networks [13-15] but somewhat less appreciated in the context of barrier tunneling [16]. We also show that the group delay and the dwell time [17], another measure of tunneling time, are identical within the slowly-varying envelope approximation. Although it has been argued strenuously by Landauer [3] that no physical significance can be attached to delays based on pulse peaks, we show, on the contrary, that they tell us the time it takes for energy to be stored and released by the barrier. While energy storage has been discussed in the context of superluminal propagation in absorbing and transparent media [18-20,16] its connection to the saturation of group delay has never been noticed.

Our model for the barrier tunneling problem is the Klein-Gordon equation which describes relativistic deBroglie waves, waves in plasmas, pulses in electromagnetic waveguides, torsion-coupled pendula, and pulse propagation in periodic dielectric structures. The versatility of the Klein-Gordon equation is indeed extraordinary. It has recently been used to analyze tunneling in electromagnetic waveguides [21-23]. Here we will derive it explicitly for a photonic bandgap structure (PBG), keeping in mind the marvelous generality of the resulting equation. A generic PBG is shown in Fig. 1. We assume a lossless, dispersionless medium with a spatially periodic refractive index of the form

$$n = n_0 + n_1 \cos(2\beta_0 z), \tag{1}$$

where $n_1 << n_0$ and β_0 is the wavenumber of the lightwave that satisfies the Bragg condition for the structure. This periodic perturbation scatters forward waves into backward

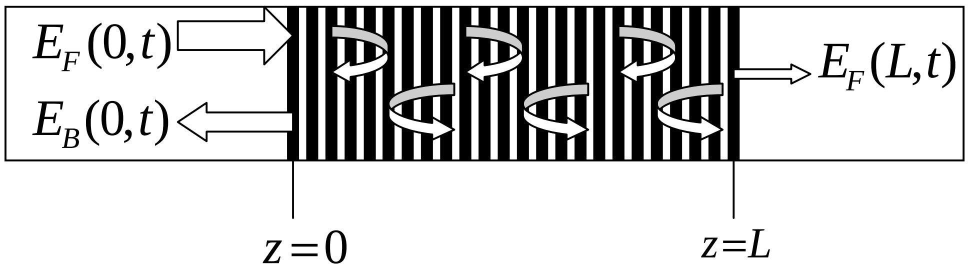

Fig. 1 Schematic of a photonic bandgap structure (PBG).

waves and hence we take the electric field within the structure as

$$\mathrm{E}(z,t) = \mathrm{Re}[E_F(z,t)e^{i(\beta_0 z - \omega_0 t)} + E_B(z,t)e^{-i(\beta_0 z + \omega_0 t)}]. \tag{2}$$

Here $E_F(z,t)$ and $E_B(z,t)$ are the envelopes of the forward and backward waves, ω_0 is the carrier angular frequency, and $\beta_0 = n_0\omega_0 / c$. Within the slowly varying envelope approximation, use of Eqs. 1 and 2 in Maxwell's equations leads to the coupled-mode equations:

$$\frac{\partial E_F}{\partial z} + \frac{1}{v}\frac{\partial E_F}{\partial t} = i\kappa E_B \tag{3a}$$

$$\frac{\partial E_B}{\partial z} - \frac{1}{v}\frac{\partial E_B}{\partial t} = -i\kappa E_F, \tag{3b}$$

where $\kappa = n_1\omega_0 / 2c$ is the coupling constant and $v = c / n_0$. By using Eq. (3a) in (3b) we obtain the Klein-Gordon equation

$$\frac{\partial^2 E_F}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 E_F}{\partial t^2} = \kappa^2 E_F. \tag{4}$$

A similar equation holds for the backward wave. It is clear then, that the coupled-mode equations are equivalent to the Klein-Gordon equation and that either can be used to describe tunneling in a variety of systems, with the proper identification of the coupling constant. We will see that the coupling constant is simply related to the cut-off frequency of the system.

For the purpose of understanding pulse evolution in the Klein-Gordon system, the simplest pulses consist of a sinusoidal envelope modulation at frequency $\Omega << \omega_0$. Then inserting $\exp[i(Kz - \Omega t)]$ in Eq. 4, we obtain the dispersion relation

$$K^2 = (\Omega^2 - \Omega_c^2) / v^2, \tag{5}$$

where $\Omega_c = \kappa v$ is the cut-off angular frequency such that K is real for $\Omega > \Omega_c$. The envelope modulation can propagate only when K is real. For $\Omega < \Omega_c$, the wavenumber becomes imaginary and the result is an exponentially attenuating evanescent "wave" of the form

#1899 - $15.00 US Received November 29, 2002; Revised December 08, 2002
(C) 2002 OSA

$$\exp(-\gamma z)\exp(-i\Omega t) \tag{6}$$

with an attenuation constant given by $\gamma = \sqrt{(\Omega_c^2 - \Omega^2)/v^2}$. This expression for an evanescent "wave" must be taken literally. It does not represent a propagating disturbance but a standing wave that simply stands and waves. Because it is a standing wave, the notion of a velocity is not relevant. Reactive energy however can be stored in such a standing wave, sloshing between its electric and magnetic forms. Where it makes contact with the outside world it can escape. Tunneling should therefore be viewed more as the dynamics of a cavity than as the propagation of a disturbance.

Since our interest is in tunneling, we assume $\Omega < \Omega_c$ and seek solutions to Eqs. 2 oscillating at frequency Ω . The envelopes satisfy

$$dE_F / dz = i\kappa E_B + i(\Omega / v)E_F , \tag{7a}$$

$$dE_B / dz = -i\kappa E_F - i(\Omega / v)E_B . \tag{7b}$$

With the boundary conditions $E_F(0) = E_0$ and $E_B(L) = 0$, the solutions are

$$E_F(z) = E_0\left[\gamma \cosh\gamma(z-L) + i(\Omega/v)\sinh\gamma(z-l)\right]/g , \tag{8a}$$

$$E_B(z) = -i[E_0\kappa \sinh\gamma(z-L)]/g , \tag{8b}$$

where $g = \gamma\cosh\gamma L - i(\Omega/v)\sinh\gamma L$. The barrier amplitude transmission coefficient is $T = E_F\left(L\right)/E_o = (\gamma/|g|)e^{i\phi_t}$, the phase of which is given by

$$\phi_t = \tan^{-1}\left[\left(\Omega/\gamma v\right)\tanh\gamma L\right] . \tag{9}$$

The reflection coefficient is $R = E_B(0)/E_0 = [\kappa\sinh(\gamma L)/|g|]e^{i\phi_r}$, whose phase is

$$\phi_r = \phi_t + \pi/2 . \tag{10}$$

The time-average stored energy within the barrier is given by

$$\left\langle U\right\rangle = \frac{1}{2}\varepsilon\int_{vol}\left[\left|E_F\right|^2 + \left|E_B\right|^2\right]dv , \tag{11}$$

which, using Eq. (8) yields

$$\left\langle U\right\rangle = \left(\frac{1}{2}\varepsilon E_0^2 A\right)\left[\frac{\kappa^2}{\gamma^2}\frac{\tanh\gamma L}{\gamma} - L\left(\frac{\Omega}{\gamma v}\right)^2 \operatorname{sech}^2\gamma L\right]\cos^2\phi_t , \tag{12}$$

where A is the cross sectional area of the barrier.

The group delay (also referred to as "phase time" in the tunneling literature) is used to characterize the occurrence in time of the peak of the tunneled pulse. It is defined as

$$\tau_g = d\phi_t / d\Omega . \tag{13}$$

By using Eq.9 in Eq. 13 we obtain the following expression for the group delay:

$$\tau_g = \frac{1}{v}\left[\frac{\kappa^2}{\gamma^2}\frac{\tanh\gamma L}{\gamma} - L\left(\frac{\Omega}{\gamma v}\right)^2 \operatorname{sech}^2\gamma L\right]\cos^2\phi_t . \tag{14}$$

For opaque barriers $(\kappa L >> 1)$, this delay is less than the "equal time" *L*/*v*, the time it would have taken for a light wave traveling with velocity *v* to traverse the distance *L*. This has led to the notion of superluminal traversal velocities with its attendant controversies regarding causality. However, in an evanescent region, the wave vector is actually imaginary. The pulse does not really travel. While velocity may not necessarily be a relevant concept in this case, a delay time certainly is. A delay time makes no assumption about the mechanism responsible for the delay while "velocity" assumes propagation, and hence, a path or trajectory. Here we will relate the delay to stored energy. We note, incidentally, that since $\kappa > \Omega / v$, the group delay cannot be negative for this structure. Indeed negative group delay is proscribed for this passive, symmetric, reciprocal structure on general grounds [15].

Comparing Eqs. (14) and (12), we find that the stored energy is proportional to the group delay and can be written

$$\langle U \rangle = \left(\frac{1}{2} \varepsilon E_0^2 A v \right) \tau_g \,. \tag{15}$$

The term in brackets in Eq. 15 can be recognized as the time-averaged incident power P_{in}. We are thus led to the result that the stored energy in the barrier is equal to the incident power times the group delay:

$$\langle U \rangle = P_{in} \tau_g \,. \tag{16}$$

The proportionality between stored energy and group delay is a result that has been proven for both lumped-element networks and distributed microwave circuits [13-15]. Basically the time required for a pulse of energy to enter an element and leave again is the average stored energy per unit incident cw power [13]. On the other hand the dwell time, a quantity introduced by Smith as a measure of the time a wavepacket spends in the barrier region, is defined as [17]

$$\tau_D = \langle U \rangle / P_{in} \tag{17}$$

and is *by definition* proportional to the stored energy. Thus we find, within the slowly-varying envelope approximation, that the group delay and the dwell time for this one-dimensional tunneling problem are identical and are proportional to the stored energy. Smith also noted that this definition of dwell time is reminiscent of the *Q* of an oscillating system in electromagnetic theory. All of this lends support to a description of the tunneling process as an excitation of a cavity that can store energy. We note that the role of energy storage has been discussed in other superluminal contexts involving absorptive or amplifying media [17,18]. There the energy is stored in the atomic excitations. Even in situations where transparent superluminal propagation has been considered, the energy has been stored in an inverted medium. In the tunneling problem considered here, the energy is stored in the electromagnetic cavity formed by the structure.

The result presented here makes it easy to understand the so-called "Hartman effect" in which the tunneling phase time becomes independent of length for thick enough barriers. From Eq. 12 we see that as $L \to \infty$, $U \to \frac{1}{2} \varepsilon E_0^2 A / \gamma$. The quantity $1/\gamma$ is the decay distance of the exponentially attenuating forward field in a very long barrier. Beyond this distance it does not matter how much more length the barrier has. The energy is all stored within the attenuation distance. Since the group delay is proportional to the stored energy, it saturates as the stored energy saturates. It should be kept in mind that true tunneling is an adiabatic process involving pulses that are long compared to the barrier width. The front of the pulse does propagate at *c* since it involves a discontinuity. However, within one transit time a standing wave is created behind the front with most of the energy stored within a decay distance of the entrance. As the bulk of the pulse, which is well behind the front, arrives at the entrance all it can do is modulate the stored energy. Both the group delay and the dwell

#1899 - $15.00 US Received November 29, 2002; Revised December 08, 2002
(C) 2002 OSA

time are a measure of the ability of the cavity to store and release energy, in a sense a figure of merit like cavity Q.

More generally, the dwell time should satisfy the relation [5]

$$\tau_D = (|R|^2\, d\phi_r / d\Omega + |T|^2\, d\phi_t / d\Omega)\,, \tag{18}$$

where $d\phi_r / d\Omega$ is the reflection delay. Since we have shown that the dwell time equals the group delay for the Klein-Gordon problem, Eq. 18 should also hold for the group delay [15]. For a lossless, symmetric barrier, $d\phi_r / d\Omega = d\phi_t / d\Omega$ and hence the transmission and reflection delays are equal. This equality of delays is what one would expect of stored energy leaking out of both ends of a symmetric structure. Since $|R|^2 + |T|^2 = 1$ for a lossless barrier, Eq. (17) reduces to

$$\tau_g = \tau_D = d\phi / d\Omega\,, \tag{19}$$

where ϕ is the phase of either the transmission or reflection coefficient. For thick barriers, since more than 99% of the output energy is in the reflected pulse, one would be better off measuring reflection delays as opposed to transmission delays. The signal-to-noise ratio would be orders of magnitude better and the result would be the same tunneling time. Of course, for a truly infinite barrier, the magnitude of the transmission coefficient is zero and hence the phase is undefined. In that case, the entire delay is associated with the reflected pulse.

In the context of barrier tunneling it is important not to make the leap from an anomalously short group delay to a superluminal propagation velocity. The delay seen here is due to energy storage and is not a propagation delay. In reality one never actually measures a velocity. What one measures are time delays and distances. Velocities are always inferred and that requires the knowledge of a trajectory and the assumption that a phenomenon indeed propagates, i.e. passes continuously through every point along the trajectory. In a situation such as the one we find for barrier tunneling, where the pulse length is much longer than the barrier length, the barrier is a lumped element with respect to the pulse envelope. In electronics it is easy enough to create delay circuits based on energy storage elements like capacitors and inductors. There the notion of a velocity does not even enter since the circuit is a lumped circuit with respect to the voltage and current waves. We suggest that a similar situation holds in barrier tunneling. Since the evanescent wave (after an initial transient) is a non-propagating field distribution, its energy is stored in the barrier and then released. Understanding the time delay in terms of energy storage frees us from controversies regarding the meaning of group velocity, causality, and the possibility of superluminal signaling.

In conclusion, we have shown that the Hartman effect, the lack of dependence of tunneling time on barrier thickness, can be explained by the saturation of stored energy with barrier length. Since the group delay is proportional to the stored energy, it saturates as the stored energy saturates. The barrier acts as an electromagnetic cavity that stores and releases energy. We believe that this description resolves a decades-old mystery and sheds further light on the nature of barrier tunneling.

Acknowledgements

The author acknowledges useful discussions with J. Eberly, R. Chiao, P. Milonni, G. Nimtz, E. Recami, and other participants of the July 2002 Mini-program on Quantum Optics, Kavli Institute for Theoretical Physics, Santa Barbara. This research was supported in part by the National Science Foundation under Grants No. PHY99-07949 and PHY01-14336.