COMMENTARY: THE PARADOX OF THE BARRIER TUNNELLING TIME OF A QUANTUM PARTICLE

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For many decades, the amount of time taken for a quantum particle to transit or tunnel through a barrier is one of the most fundamental topics in physics debated by many scientists worldwide. In a certain sense, time is one of the most fundamental parameters in the classical world we are living in now and time can be easily measured by using a stopwatch. However, in the quantum regime, the definition of the measure of time or the concept of the "clock" is a more complex idea to be understood [1].

Classically, it is easy to understand the concept of time by using an external device such as a stopwatch to measure the time of a process occurring. However, a unified concept of a quantum clock does not exist and the classical concept of time breaks down in the quantum limit. Quantum-mechanically the particle of certain energy has probably transverse a barrier and no one cannot be sure that the time recorded by using the stopwatch (or classical clock) is correct.

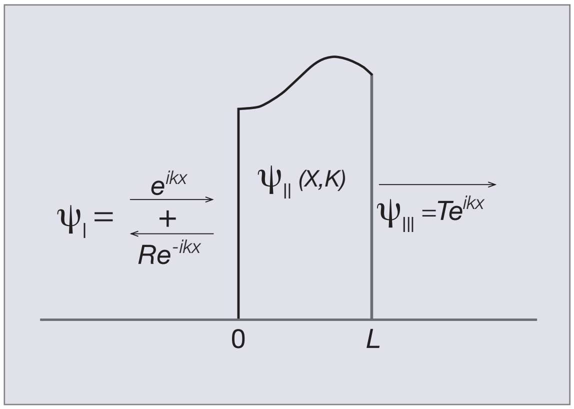

Fig. 1 - Schematic of the barrier tunnelling problem [2]

Previous attempts to observe the time evolution of certain physical quantities are based on their own set of quantum tunnelling time scales, and these time scales may not be compatible with each other and the classical concept of time. Thus, it seems that time in the quantum scale is very much open to the individual's interpretation instead of being a fundamental parameter as defined in the classical world.

Despite the various different definitions of tunnelling time, there are two – delay time and dwell time – which are more established than others even though these two quantum time scale cannot be related to the classical time. Nevertheless, the two time scales are used here to illustrate the paradox of the superluminality (faster than speed of light) property of a barrier tunnelling quantum particle.

It has been shown theoretically and experimentally that the delay and dwell time (τ_D) in tunnelling through a thick barrier (which is normally assumed to be equivalent to the propagation time required for a quantum particle to tunnel through a barrier) is independent of the barrier width (L) where the barrier is shown in Fig. 1. If L increases, τ_D remains constant and thus we will reach a point where the corresponding tunnelling velocity ($v = L/\tau_D$) of the quantum particle using these time scales is shown to exceed the speed of light and becomes unlimited when the transmission coefficient of the barrier approaches zero (i.e. the opaque limit). In the opaque barrier limit, is the quantum particle really able to travel faster than speed of light as it tunnel through a thick barrier as inferred? This sounds suspicious and the argument has aroused repeated attacks on the various quantum tunnelling time scales used.

This paradox has baffled numerous physicists for many decades until Professor Herbert G. Winful's landmark paper which refutes the common assumption that τ_D (or any other quantum time scales used) is equivalent the propagation time of a quantum particle through a barrier [2]. He proved mathematically that the delays in barrier tunneling are actually describing the outflow of energy from both ends of the barrier by treating quantum particles as a wave packet and we can use a simple analogy to relate the delay experienced by the quantum particle during the tunnelling process (in terms of energy outflow) to real life examples.

For instance, there are two similar trains with a carrying capacity of 20,000 passengers: Train 1 carries 10,000 passengers and Train 2, 1,000 passengers. Train 1 travels from Singapore to Kuala Lumpur (analogous to barrier width L_1) and Train 2 travels from Singapore to Thailand Border (barrier width $L_2 > L_1$) without stopping. Apparently, Train 1 will reach its destination, Kuala Lumpur, earlier if both trains set off at the same time ($\tau_{train1} < \tau_{train2}$) as shown in Fig. 2. However, if we now define the arrival time (energy outflow time) as the average time for each passenger (energy unit) to travel from Singapore to their respective destination and exit the train station, the passengers from Train 1 will apparently take more time to exit the train station than those in Train 2 ($\tau_{exit1} > \tau_{exit2}$) and assuming that the time the 10,000 passengers in Train 1 take to exit the Kuala Lumpur train station is the same as the time the 1,000 passengers in Train 2 need to travel from Kuala Lumpur to Thailand Border and exit the Thailand Border train station then we will find that the passengers from both trains will take the same amount of time to reach their destination and exit their respective train stations (i.e. the delay time τ_D is the same even though barrier width is different).

Thus, this explains why there is a saturation of τ_D, which is basically the energy outflow time, when the barrier is thick. The limit imposed is due to the amount of energy that can be stored in the barrier (i.e. similar to the number of passengers who has boarded the train) even when the barrier width (distance between Singapore to the destination train station) increases indefinitely.

For quantum tunnelling across a barrier, we assume that barriers were erected at the some of the train cabins' entrances in Train 2, therefore, only 1,000, instead of 20,000, passengers managed to get on the train (analogous to the reflection of a huge fraction of energy of the wave packet in the case of a thick barrier). In contrast, all the 20,000 passengers can board the train if the region is barrier-free (100% transmission). Thus, we can see that the barrier reduces the amount of energy that can be stored and the delay time has to be measured with respect to the stored energy under the barrier.

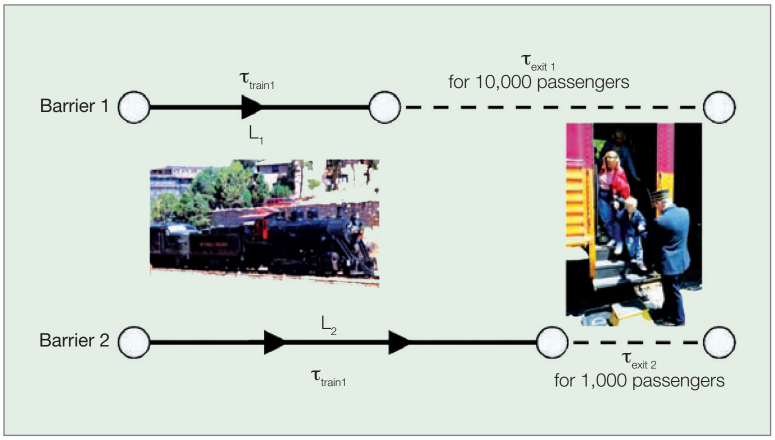

Fig. 2 - Analogy of the train and barrier tunnelling time

Some immediate applications of Professor Herbert G. Winful's theory are currently in photonic band gap structures and cavities. However, it is expected that the same theory can be applied to describe the rate of energy loss to the quantum-mechanical tunnelling of electrons in field emission too.

Note: Professor Winful has showed a very concise and clear explanation of his mathematical treatment of the problem and the reader is advised to go through Refs. [2] for more details.

Refs:

[1] Christian Bracher, Manfred Kleber, and Mustafa Riza, Phys. Rev. A, **60**, 1864 (1999)

[2] Herbert G. Winful, Optics Lett., **10**,1491 (2002); Phys. Rev. Lett., **91**, 260401 (2003)