Relation between quantum tunneling times for relativistic particles

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(Received 29 June 2004; published 23 November 2004)

A general relation between the phase time (group delay) and the dwell time is derived for relativistic tunneling particles described by the Dirac equation. It is shown that the phase time equals the dwell time plus a self-interference delay which is a relativistic generalization of previous results.

DOI: 10.1103/PhysRevA.70.052112 PACS number(s): 03.65.Xp, 73.40.Gk, 73.43.Jn

I. INTRODUCTION

The time it takes for a particle or wave packet to tunnel through a potential barrier has been debated for decades [1–8]. The fact that there is a finite duration for the tunneling process is not in doubt. The debate centers around the validity of the various proposed tunneling times, the relation between those times, and the physical meaning of a tunneling time especially when it predicts apparent superluminal velocities. Two of the more commonly used tunneling times are the dwell time (defined through the integrated probability density under the barrier) and the phase time (defined by the energy derivative of the transmission phase shift). These two times, however, are not entirely unrelated. They are equal under certain circumstances but generally differ as a result of quantum interference effects [9–11]. Both times also saturate with increasing barrier length, a phenomenon known as the Hartman effect [2] and which has been claimed to lead to infinite tunneling velocities for opaque barriers [6]. This effect has recently been explained as arising from the saturation of stored energy or number of particles under the barrier [11–13]. With some exceptions [14–23], most discussions of quantum tunneling time have been based on the nonrelativistic Schrödinger equation even when apparent "faster than *c*" effects are considered. In particular there has been no discussion of the relation between the various tunneling times for relativistic particles. Here we derive an exact relation between the phase time and the dwell time for relativistic particles that satisfy the Dirac equation. We show by means of an explicit stationary state calculation that the phase time is equal to the dwell time plus a self-interference delay which is a relativistic generalization of earlier results.

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II. RELATIVISTIC ONE-DIMENSIONAL SCATTERING RELATIONS

We begin with a Dirac particle of mass m and total energy E traveling in one dimension in the presence of a real potential barrier or potential well $V(z)$ that occupies the region $0<z<L$. The one-dimensional relativistic equation that describes its stationary state is

$$H_0\psi = [-i\hbar c\alpha_z\partial_z + \beta mc^2 + V(z)]\psi = E\psi, \tag{1}$$

where $\alpha_z=\begin{pmatrix}0 & \sigma_z\\ \sigma_z & 0\end{pmatrix}$ and $\beta=\begin{pmatrix}I & 0\\ 0 & -I\end{pmatrix}$ are the well known 4×4 Dirac matrices specified in terms of the Pauli matrices and the unit matrix **I**. Each element of α_z and β is understood to be a 2×2 matrix. In Eq. (1) the electron is represented by the relativistic wave packet described by the Dirac spinor wave function

$$\psi = \begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{pmatrix}. \tag{2}$$

Our goal is to obtain a general relation between dwell and phase times for the one-dimensional scattering problem. To that end, we first take a derivative of the Dirac equation with respect to E

$$[-i\hbar c\alpha_z\partial_z + \beta mc^2 + V(z) - E](\partial\psi/\partial E) = \psi. \tag{3}$$

Then the Hermitian conjugate of Eq. (1) is multiplied from the right by $\partial\psi/\partial E$, yielding

$$\{i\hbar c\partial_z\psi^\dagger\alpha_z + \psi^\dagger\beta mc^2 + [V(z) - E]\psi^\dagger\}(\partial\psi/\partial E) = 0. \tag{4}$$

Equation (3) is then left-multiplied by $\psi^\dagger$ and added to Eq. (4), resulting in the expression

$$-i\hbar c\left(\psi^\dagger \alpha_z \frac{\partial^2\psi}{\partial E \partial z} + \frac{\partial \psi^\dagger}{\partial z}\alpha_z \frac{\partial \psi}{\partial E}\right) = -i\hbar c \frac{\partial}{\partial z}\left(\psi^\dagger \alpha_z \frac{\partial \psi}{\partial E}\right) = \psi^\dagger \psi. \tag{5}$$

Integrating over the barrier from 0 to L we find

$$\left(\psi^\dagger \alpha_z \frac{\partial \psi}{\partial E}\right)_{z=L} - \left(\psi^\dagger \alpha_z \frac{\partial \psi}{\partial E}\right)_{z=0} = \frac{i}{\hbar c}\int_0^L \psi^\dagger \psi \, dz. \tag{6}$$

Equation (6) is a general theorem that describes the sensitivity of the wave function to variations in energy and allows us to relate the various tunneling times in a straightforward manner.

The wave function ψ is a solution of the stationary state scattering problem. In front of the barrier (region I) it consists of incident and reflected waves:

$$\psi_I = \begin{pmatrix} 1 \\ 0 \\ \eta \\ 0 \end{pmatrix} e^{ikz} + R \begin{pmatrix} 1 \\ 0 \\ -\eta \\ 0 \end{pmatrix} e^{-ikz}, \tag{7}$$

where $\eta(k) \equiv \hbar ck/(E+mc^2)$ and $\hbar ck = \sqrt{E^2 - m^2c^4}$. The amplitude reflection coefficient is $R = |R|e^{i\phi_r}$. The transmitted wave behind the barrier (region III) is given by

$$\psi_{III} = T \begin{pmatrix} 1 \\ 0 \\ \eta \\ 0 \end{pmatrix} e^{ikz}, \tag{8}$$

with $T = |T|e^{i\phi_t}$ being the transmission coefficient. With these wave functions the left side of Eq. (6) can be evaluated to yield

$$i2\eta\left(|T|^2 \frac{d\phi_0}{dk} + |R|^2 \frac{d\phi_r}{dk} + \frac{\mathrm{Im}(R)}{\eta}\frac{d\eta}{dk}\right)\frac{\partial k}{\partial E} = \frac{i}{\hbar c}\int_0^L \psi^\dagger \psi \, dz, \tag{9}$$

where $\phi_0 = \phi_t + kL$ is the total phase of the transmitted wave and we have used the fact that $|R|^2 + |T|^2 = 1$ for a lossless barrier.

The dwell time is defined as [24,25]

$$\tau_d = \frac{\int_0^L \psi^\dagger \psi \, dz}{j_{in}}, \tag{10}$$

where $j_{in} = c\psi^\dagger \alpha_z \psi = 2\hbar kc^2/(E+mc^2)$ is the probability current density for incident particles. The phase time or group delay in transmission is [26,27]

$$\tau_{gt} = \hbar \frac{d\phi_0}{dE}, \tag{11}$$

while the reflection group delay is

$$\tau_{gr} = \hbar \frac{d\phi_r}{dE}. \tag{12}$$

Equation (9) shows that the exact relation between these times is

$$\tau_d = |T|^2 \tau_{gt} + |R|^2 \tau_{gr} - \tau_i, \tag{13}$$

where we have defined

$$\tau_i = -\frac{\mathrm{Im}(R)}{\eta}\hbar \frac{\partial \eta}{\partial E} = -\frac{m}{\hbar k^2}\,\mathrm{Im}(R) = -\frac{\mathrm{Im}(R)\hbar}{2E_k(1 + E_k/2mc^2)} \tag{14}$$

as the self-interference delay. Here $E_k = E - mc^2$. Each of these times has a well defined meaning. The dwell time is the mean time spent by an incident particle of energy E in the barrier region $0 < z < L$ regardless of whether it is ultimately reflected or transmitted. The transmission group delay is the time it takes for the peak of the transmitted wave packet to appear at $z = L$, measured from the moment the peak of the incident wave packet would have reached the input $z = 0$ in the absence of reflections. Similarly, the reflection group delay is the time it takes for the peak of the reflected wave packet to appear at $z = 0$, measured from the moment the peak of the incident wave packet would have reached the input $z = 0$ in the absence of reflections. Both of these group delays are extrapolated delays assuming a freely propagating incident wave packet. In reality, the incident wave packet is perturbed as a result of interaction with the barrier during its approach. The self-interference delay is an extra delay that arises from the fact that the wave packet in front of the barrier is composed of incident and reflected components which interfere with each other. It is related to the difference between the time the *extrapolated* peak of the incident wave packet arrives at the input and the actual time the peak of the overall packet arrives at the same point. Because true tunneling is a quasistatic phenomenon that requires that the wave packet spatial extent be much greater than the barrier width, an incident wave packet will always interfere with itself in front of the barrier. This self-interference delay is greatest at low values of incident kinetic energies when the particle spends most of its time caught up in a standing wave in front of the barrier. This delay vanishes at transmission resonances where the reflectivity is zero and at values of the reflection phase equal to $n\pi$, $n = 0, 1, 2, \ldots$. A term similar to this has also been found in the tunneling of electromagnetic waves through constricted waveguides [28] and photonic band gap structures [29]. In Eq. (14) the term $E_k/2mc^2$ in the denominator is a relativistic correction to the form of the self-interference delay. The nonrelativistic form is obtained in the limit $E_k \gg E - mc^2$. In that limit, the self-interference delay is analogous to the $\hbar/2E$ term found by Smith to be a quantum resonance correction to the collision lifetime in scattering theory [24].

Although we have used the terminology potential "barrier" in this discussion, the general results above also hold for potential wells. Also, for a symmetric potential $\tau_{gt} = \tau_{gr} \equiv \tau_g$, and hence the relation between tunneling times reduces to

$$\tau_g = \tau_d + \tau_i. \tag{15}$$

We note that there is yet another tunneling time scale known as the Larmor time which measures the total angular change of a particle with spin tunneling in the presence of a magnetic field [25]. This Larmor time has been shown to equal the dwell time for both relativistic and nonrelativistic particles [19,20,25].

It is often stated that tunneling times for reflection and transmission must satisfy the relation [4]

$$\tau_d = |T|^2\tau_t + |R|^2\tau_r. \tag{16}$$

This relation has been used as a criterion for the validity of various tunneling time definitions [4]. It has also been criticized as being inappropriate for describing quantum events [30]. Here we see that this much quoted criterion cannot be the sole arbiter for the correctness of a tunneling time definition. While it is indeed possible to define certain times τ_t and τ_r that satisfy the relation (16), there is no *a priori* reason why this relation should generally hold for quantum mechanical scattering events. Indeed, if τ_t and τ_r are taken as the phase times in transmission and reflection, then the correct relation between those times and the dwell time is the one given by Eq. (13). The tunneling process is a wave phenomenon and must generally include interference contributions. For the phase times the relation (16) holds only in the classical regions and at transmission resonances where self-interference effects can be neglected.

III. TUNNELING TIMES FOR A RECTANGULAR BARRIER

We test the general relations obtained above by applying them to a rectangular potential barrier described by $V(z) = V_0\Theta(z)\Theta(L-z)$. The relativistic interaction dynamics of a particle with total energy E incident on a barrier with height V_0 can be divided into three cases. In the case that the potential barrier is low enough to satisfy $V_0 < E - mc^2 = E_k$, the particle has enough energy to propagate over the potential barrier. Also, it can be shown that when the potential barrier is strong enough to satisfy $V_0 > E + mc^2$ it can become supercritical and spontaneously emit positrons or electrons. This is a transient phenomenon that takes us into the realm of the Klein tunneling [31] which has no nonrelativistic equivalent. We therefore restrict our study to the case in which the potential barrier height satisfies $E - mc^2 < V_0 < E + mc^2$. We will also restrict our discussion to relativistic positive energy Dirac particles. When the barrier height V_0 satisfies $E - mc^2 < V_0 < E + mc^2$ the wave function within the barrier (region II) is given by [32,21,14]

$$\psi_{II} = C\begin{pmatrix} 1 \\ 0 \\ \dfrac{i\hbar\kappa c}{E - V_0 + mc^2} \\ 0 \end{pmatrix} e^{-\kappa z} + D\begin{pmatrix} 1 \\ 0 \\ \dfrac{i\hbar\kappa c}{E - V_0 + mc^2} \\ 0 \end{pmatrix} e^{\kappa z}, \tag{17}$$

where $\hbar c\kappa = \sqrt{m^2c^4 - (V_0 - E)^2}$, κ being the decay constant for the evanescent wave within the barrier. The coefficients C, D, R, and T are obtained by imposing continuity of the wave function across the interfaces $z=0$, L and are given by [21,14]

$$C = (1 - i\xi)e^{\kappa L}/2\gamma, \tag{18a}$$

$$D = (1 + i\xi)e^{-\kappa L}/2\gamma, \tag{18b}$$

$$T = e^{-ikL}/\gamma, \tag{19}$$

$$R = -i[(\xi + 1/\xi)\sinh 2\kappa L]/2\gamma, \tag{20}$$

where $\xi \equiv (k/\kappa)(E - V_0 + mc^2)/(E + mc^2)$ and $\gamma = \cosh \kappa L - (i/2)(\xi - 1/\xi)\sinh \kappa L$.

We have two different methods to calculate the phase time. One is by taking the energy derivative of the transmission phase. The other is by calculating the dwell time and adding the self-interference delay. With the use of the wave function inside the barrier ψ_{II} Eq. (17), the evaluation of the dwell time yields

$$\tau_d = \frac{L}{2\kappa\hbar c^2 f^2 \xi}\left(mc^2(1 + \xi^2)\frac{\sinh 2\kappa L}{2\kappa L} + (E - V_0)(1 - \xi^2)\right), \tag{21}$$

where $f = |\gamma|$. From the expression for the reflection coefficient we find that the self-interference term, using Eq. (14), is

$$\tau_i = \frac{mc^2(1 + \xi^2)}{4\hbar c^2 f^2 k^2 \xi}\sinh 2\kappa L. \tag{22}$$

The sum of these two times is

$$\tau_g = \tau_d + \tau_i = \frac{L}{2\kappa\hbar c^2 f^2 \xi}\left((1 + \xi^2)\frac{\sinh 2\kappa L}{2\kappa L}\frac{mV_0(2E - V_0)}{\hbar^2 k^2} + (1 - \xi^2)(E - V_0)\right). \tag{23}$$

On the other hand the total transmission phase shift is

$$\phi_0 = \arg(T) + kL = \tan^{-1}\left[\frac{1}{2}\left(\xi - \frac{1}{\xi}\right)\tanh \kappa L\right]. \tag{24}$$

The energy derivative of the transmission phase shift ϕ_0 yields the same expression as Eq. (23), in agreement with previous work [14,18,21]. Thus the group delays calculated by these two very different methods are in complete agreement, confirming the relation of Eq. (15) between phase time, dwell time, and self-interference delay.

Figure 1 shows these three times plotted as a function of normalized kinetic energy. The times are normalized by $\tau_0 = L/c$, the transit time for a particle traveling at the speed of light. It is seen that the normalized group delay can be less than 1 for kinetic energies less than V_0. This should not be taken to mean that the wave packet peak traveled as a rigid entity from input to output faster than the speed of light. As a matter of fact, the peak does not travel from input to output, as has been confirmed in direct numerical simulations of relativistic wave equations [13,17,33]. The dwell time is the $1/e$ lifetime of particles escaping out of both ends of the

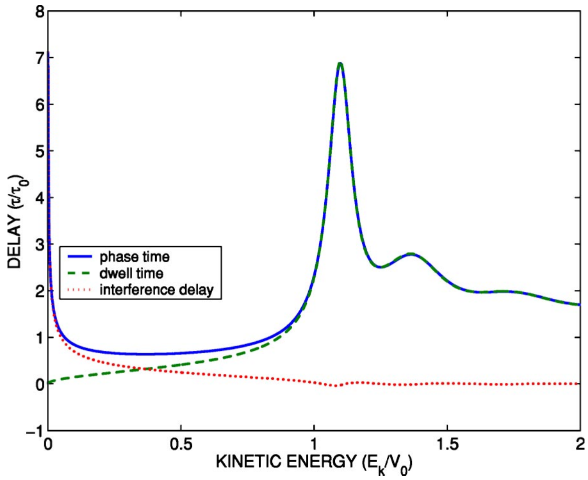

FIG. 1. Phase time (solid curve), dwell time (dashed curve), and self-interference delay (dotted curve) versus normalized kinetic energy. Here $V_0L/\hbar c=2\pi$ and $V_0/mc^2=0.5$.

barrier. To that we add the time spent waiting in front of the barrier, the self-interference delay. The total is the phase time. For energies above the barrier ($E_k > V_0$) the dwell time and phase time are identical, but for the region $E_k < V_0$ there is a difference between the two because of the self-interference delay. At low energies, as the kinetic energy goes to zero the dwell time, the time spent in the barrier, goes to zero. The self-interference delay however goes to infinity. This means the particle does not even enter the barrier but is trapped in a standing wave in front of the barrier. We examine more closely the effect of relativistic corrections on the delay times in Fig. 2. The strength of relativistic effects is determined by the ratio $\nu \equiv V_0/mc^2$. In the nonrelativistic limit $\nu \ll 1$, the self-interference delay becomes equal to the dwell time when $E_k=V_0/2$. The three times display a certain symmetry about $E_k=V_0/2$. In the tunneling regime, the dwell time for relativistic particles is generally longer than that for nonrelativistic particles.

The Hartman effect is the saturation of the group delay with barrier length. In the limit that $L\to\infty$, the probability density inside the barrier is proportional to the decaying exponential $|\psi|^2 \propto e^{-2\kappa L}$. The integrated probability saturates with barrier length and hence the dwell time, self-interference, and phase delay saturate. As $L\to\infty$ we find

$$\tau_d = \frac{2m}{\hbar\kappa^2}\frac{\xi}{1+\xi^2}, \quad \tau_i = \frac{2m}{\hbar k^2}\frac{\xi}{1+\xi^2},$$

$$\tau_g = \tau_d + \tau_i = \frac{2m}{\hbar}\left(\frac{1}{k^2}+\frac{1}{\kappa^2}\right)\frac{\xi}{1+\xi^2}. \quad (25)$$

The fact that these times saturate with length indicates that they are not propagation delays. To accept them as propagation delays would imply that the particle somehow knows that the barrier has been made longer and hence must increase its speed by just the right amount to cover the greater distance in the same amount of time. Such a state of affairs is clearly untenable. As we have previously shown, for distortionless tunneling the wave packet has to be much

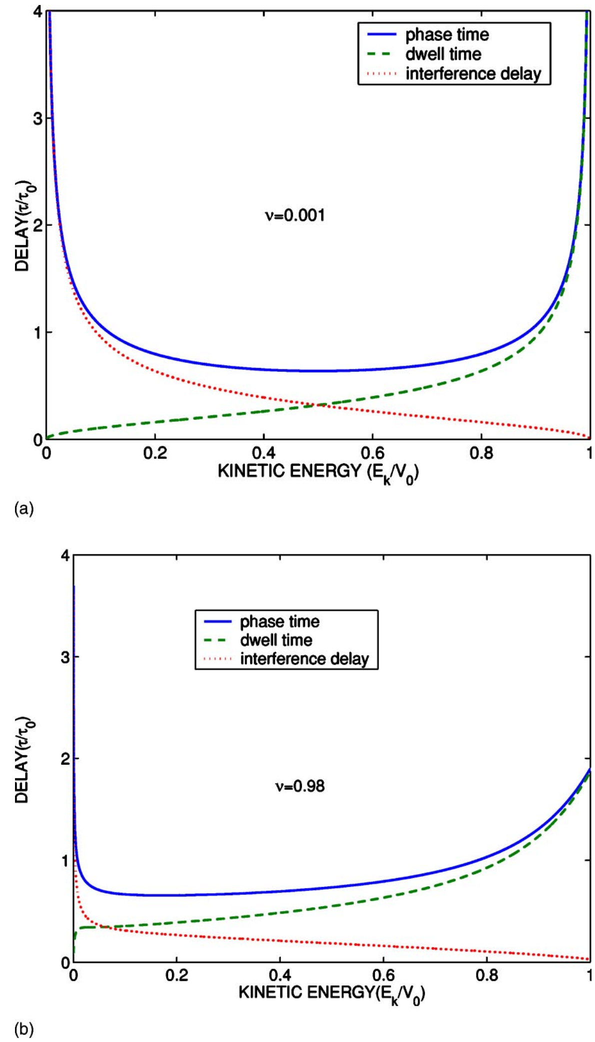

FIG. 2. Phase time (solid curve), dwell time (dashed curve), and self-interference delay (dotted curve) versus normalized kinetic energy. (a) The nonrelativistic limit; here $V_0L/\hbar c=2\pi$ and $V_0/mc^2=0.001$. The relativistic limit; here $V_0L/\hbar c=2\pi$ and $V_0/mc^2=0.99$.

longer than the barrier [13,33]. The uncertainty Δz in the location of the particle is much greater than the width L of the barrier. Similarly, the phase time or group delay τ_g is much smaller than the duration Δt of the wave packet, the time it takes for the wave packet to pass a given point. While it is interacting with the barrier we cannot say anything about where it is. We can only say where the particle is after it has completed its interaction with the barrier. The duration of that interaction is the length of the wave packet. It is only after the completed scattering event can we say whether a particle has been reflected or transmitted. Thus, a more meaningful "tunneling time" might be one determined by the length of the wave packet.

We now show that in the nonrelativistic limit we recover previous results for the tunneling times. We do this by "slowing down" the particle, assuming that the speed of the incident particle is much less than c, such that its kinetic energy $E_k=E-mc^2 \ll mc^2$ and that also means that the barrier height $V_0 \ll mc^2$. In these limits ξ reduces to $\xi=k/\kappa$, where $k \approx \sqrt{2mE_k}/\hbar$ and $\kappa \approx \sqrt{2m(V_0-E_k)}/\hbar$. Inserting these terms into the tunneling time expressions we obtain

$$\tau_i = \frac{mL}{2\hbar k f^2}\left(1 + \frac{\kappa^2}{k^2}\right)\frac{\sinh 2\kappa L}{2\kappa L}, \qquad (26)$$

$$\tau_d = \frac{mL}{2\hbar k f^2}\left(1 + \frac{k^2}{\kappa^2}\right)\frac{\sinh 2\kappa L}{2\kappa L} + \left(1 - \frac{k^2}{\kappa^2}\right), \qquad (27)$$

$$\tau_g = \tau_d + \tau_i = \frac{mL}{2\hbar k f^2}\left(\frac{k}{\kappa} + \frac{\kappa}{k}\right)\frac{\sinh 2\kappa L}{2\kappa L} + \left(1 - \frac{k^2}{\kappa^2}\right). \qquad (28)$$

These times agree with previously derived nonrelativistic expressions for the self-interference delay [11], the dwell time [25], and the phase time [2].

IV. CONCLUSION

In this paper we have investigated the delay times involved in relativistic quantum tunneling using Dirac's theory. We have derived tunneling times for the Dirac particles and established that the phase time is equal to the dwell time plus a self-interference delay which is dependent on the dispersion outside the barrier region. All three times saturate with barrier length, a manifestation of the Hartman effect and explainable by the saturation of the probability density or number of particles in the barrier region. In the nonrelativistic limit we recover previous results relating the phase time and dwell time based on the Schrödinger equation.

ACKNOWLEDGMENT

This work was partially supported by the NSF through the FOCUS Center (PHY-0114336).

APPENDIX: THE KLEIN TUNNELING REGIME

The key result of this work [as given in Eqs. (13) and (14)] formally holds regardless of the strength of the barrier, so long as the single-particle Dirac equation remains a valid description of the interaction. For strong barriers such that $V_0 > E + mc^2$ we encounter the phenomenon of Klein tunneling [31] which has no equivalence in the nonrelativistic Schrodinger equation. Here the particle is able to tunnel through the barrier without attenuation, a process mediated by spontaneous particle-antiparticle pair production [31,34]. Elsewhere we investigate the detailed dynamics of this highly relativistic process. For now we present the expressions for the dwell time, phase time, and self-interference delay in this regime and show that they satisfy Eqs. (13) and (14).

The wave functions inside the barrier are now propagating waves with wave number κ' defined by $\hbar c\kappa' = \sqrt{(V_0 - E)^2 - m^2c^4}$. The reflection coefficient is now

$$R = [i(\xi' - 1/\xi')\sin 2\kappa' L]/4\gamma', \qquad (A1)$$

where $\xi' \equiv (k/\kappa')(V_0 - E - mc^2)/(E + mc^2)$, and $\gamma' = \cos \kappa' L + (i/2)(\xi' + 1/\xi')\sin \kappa' L$.

Using Eq. (14) we find that the self-interference delay is

$$\tau_i = \frac{mc^2(1/\xi' - \xi')\sin 2\kappa' L}{4\hbar c^2 k^2 f'^2}, \qquad (A2)$$

where $f' = |\gamma'|$. To confirm Eq. (15) we now calculate the self-interference delay by taking the difference between the phase delay and the dwell time given by [21]

$$\tau_g = \frac{L}{2\kappa'\hbar c^2 f'^2 \xi'}\left((1 + \xi'^2)(V_0 - E) - (1 - \xi'^2)\frac{mV_0(2E - V_0)}{\hbar^2 k^2}\frac{\sin 2\kappa' L}{2\kappa' L}\right), \qquad (A3)$$

$$\tau_d = \frac{L}{2\kappa'\hbar c^2 f'^2 \xi'}\left((V_0 - E)(1 + \xi'^2) - mc^2(1 - \xi'^2)\frac{\sin 2\kappa' L}{2\kappa' L}\right). \qquad (A4)$$

Here again we find that

$$\tau_i = \tau_g - \tau_d = \frac{mc^2(1/\xi' - \xi')\sin 2\kappa' L}{4\hbar c^2 k^2 f'^2}.$$

Clearly, the self-interference delay vanishes at the transmission resonances $(\kappa' L = n\pi/2)$ and where $\xi'^2 = 1$.

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