Stability of phase locking in coupled semiconductor laser arrays

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It is shown that amplitude phase coupling (as described by the linewidth enhancement factor α) leads to unstable phase locking in semiconductor laser arrays with evanescent coupling.

The problem of synchronization of coupled oscillators is fundamental to many areas of science.[1] When synchronization is successful the several oscillators act as one, with a unique frequency and with well-defined phase relationships between the oscillators. When synchronization fails the behavior of the oscillators is marvelously complex. Here beat oscillations, quasi-periodic motions, and chaotic dynamics are the norm. Phase-locked semiconductor laser arrays are an important example of the complicated dynamics of coupled nonlinear oscillators. To date, however, there have been few experiments with the temporal resolution necessary to observe the interesting dynamics.[2-4] We recently presented numerical simulations which showed that the irregular and undamped spiking behavior observed by Elliott *et al.*[2] is intrinsic to the laser arrays themselves.[5] The temporal stability of semiconductor laser arrays appears questionable, at best. In this letter we present a linear stability analysis that shows that laser arrays are unstable over large regions of a parameter space spanned by the coupling constant and the injection current. We point out the role of amplitude phase coupling, as described by the linewidth enhancement factor[6] α, in destabilizing phase locking between adjacent laser elements.

The dynamic behavior of semiconductor laser arrays with evanescent coupling is described by the equations[5]:

$$\frac{dE_i}{dt} = \frac{g'}{2}(N_i - N_{th})E_i - \frac{\kappa c}{n}[E_{i+1}\sin(\phi_{i+1} - \phi_i) + E_{i-1}\sin(\phi_{i-1} - \phi_i)], \tag{1}$$

$$\frac{d\phi_i}{dt} = -\frac{\alpha g'}{2}(N_i - N_{th}) + \frac{\kappa c}{n}\left[\frac{E_{i+1}}{E_i}\cos(\phi_{i+1} - \phi_i) + \frac{E_{i-1}}{E_i}\cos(\phi_{i-1} - \phi_i)\right], \tag{2}$$

$$\frac{dN_i}{dt} = P - \frac{N_i}{\tau_s} - \left(\frac{1}{\tau_p} + g'(N_i - N_{th})\right)E_i^2, \tag{3}$$

where N_i is the carrier density, E_i is the amplitude, and ϕ_i is the phase of the electric field in the ith channel. The other parameters are the differential gain g' (suitably reduced by a mode confinement factor), the coupling constant κ, the threshold carrier density N_{th}, the linewidth enhancement factor α, the pump rate P, the photon lifetime τ_p, and the spontaneous carrier lifetime τ_s. The phase-locked state, if it exists, is found by setting all the time derivatives to zero and solving for the steady-state values of E_i, ϕ_i, and N_i. In what

follows we show that for a wide range of parameters, the steady-state phase-locked state is unstable.

A stability analysis of Eqs. (1)–(3) is a daunting task for large N. For $N = 2$, however, it is possible to obtain closed-form expressions for the stability boundaries. For convenience we introduce the following rescaled variables and parameters:

$$X_i = (\tfrac{1}{2}g'\tau_s)^{1/2}E_i, \quad Z_i = (1/2)g'N_{th}\tau_p(N_i/N_{th} - 1)$$

$$p = (1/2)g'N_{th}\tau_p(P/P_{th} - 1), \quad \eta = (\kappa c/n)\tau_p,$$

$$T = \tau_s/\tau_p.$$

Then, for a two-element array, the coupled mode equations become

$$\dot{X}_1 = Z_1 X_1 - \eta X_2 \sin\theta, \tag{4a}$$

$$\dot{X}_2 = Z_2 X_2 + \eta X_1 \sin\theta, \tag{4b}$$

$$\dot{\theta} = -\alpha(Z_2 - Z_1) + \eta(X_1/X_2 - X_2/X_1)\cos\theta, \tag{4c}$$

$$T\dot{Z}_1 = p - Z_1 - (1 + 2Z_1)X_1^2, \tag{4d}$$

$$T\dot{Z}_2 = p - Z_2 - (1 + 2Z_2)X_2^2, \tag{4e}$$

where $\theta = \phi_2 - \phi_1$ and the dots signify derivatives with respect to a reduced time t/τ_p. Equations (4) possess the equilibria:

$$(I)\, X_1 = X_2 = \sqrt{p}, \quad Z_1 = Z_2 = 0, \quad \theta = 0, \tag{5a}$$

$$(II)\, X_1 = X_2 = \sqrt{p}, \quad Z_1 = Z_2 = 0, \quad \theta = \pi. \tag{5b}$$

To investigate the stability of these steady states we introduce small perturbations and linearize Eqs. (4) about their steady-state values. The Routh–Hurwitz criterion is used to determine the regions of parameter space in which the steady-state solutions are stable. We find that the out-of-phase (II) solution is stable under the condition

$$\eta < (1 + 2p)/2\alpha T, \tag{6}$$

while the in-phase (I) solution is stable for

$$\eta > \alpha p/(1 + 2p). \tag{7}$$

Figure 1 shows the instability domain in the plane of the variables η (the coupling strength) and p (the normalized excess pump current). The other key parameters are $\alpha = 5$ and $T = 2 \times 10^3$. It is clear that the phase-locked state is unstable over a wide region of the η-p plane. It should be noted that experimental values of η are in the range 10^{-3}–10^{-4} and that a value of $p = 0.05$ corresponds to a pump current of order 1.3 times the threshold value. It is thus obvious that stability is assured only for very small values of η in which case the lasers are essentially independent, or for
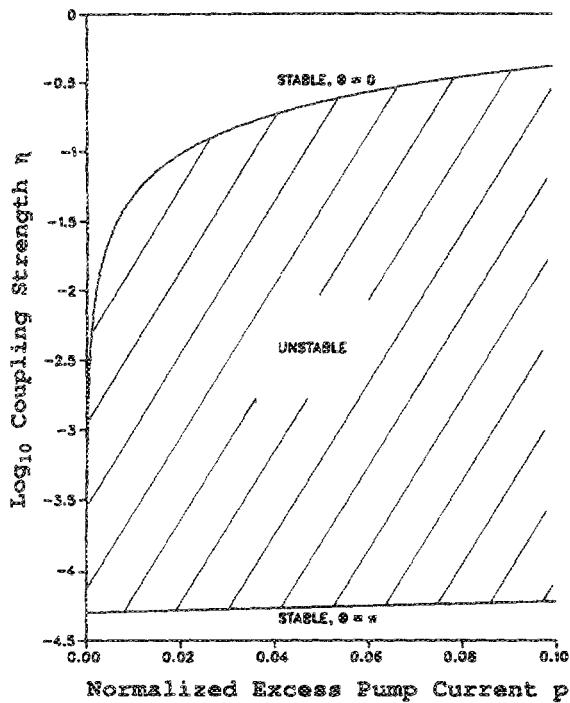
FIG. 1. Instability domain in the plane of the variables $\eta = \kappa c \tau_p / n$, the coupling strength and $p = (1/2) g' N_{th} \tau_p (P/P_{th} - 1)$, the normalized excess pump current.

very large values of η, in which case the two lasers are so tightly coupled that they act as one.

In the unstable regime the laser array exhibits sustained oscillations which may be periodic, quasi-periodic, or chaotic, depending on the values of the chosen parameters. Figure 2 shows time series of the amplitude and relative phase in the unstable region for $p = 0.05$ and $\eta = 10^{-4}$. The initial conditions were $X_1 = X_2 = \sqrt{p}$, $Z_1 = Z_2 = 0$, and $\theta = 0.1$. Note that the phases do not lock to a constant value. The phase difference oscillates about a mean value of $\theta = \pi$. Far-field measurements taken with averaging detectors will reveal a twin-lobed structure usually associated with out-of-phase locking. However, our analysis shows that the phases are not necessarily locked. It is important to note also that our analysis does not make any assumptions concerning the relative losses of the in-phase and out-of-phase modes. The preference of the system to operate (on the average) in the out-of-phase mode is intrinsic to the nature of evanescent coupling and may be explained on the basis of a maximum emission principle. Near the stability boundaries, the linearized equations for the fluctuations yield an oscillation frequency given by [7]

$$\omega = \left[\frac{1 + g' N_0 \tau_p}{\tau_p \tau_s} \left(\frac{I}{I_{th}} - 1 \right) + \left(\frac{\kappa c}{n} \right)^2 \right]^{1/2}, \quad (8)$$

where I is the injection current and $N_0 = N_{th} - 1/g' \tau_p$. For small κ, the pulsation frequency has the characteristic dependence $(I/I_{th} - 1)^{1/2}$ associated with the relaxation oscillation resonance. However, when κ is large, the pulsation frequency becomes simply the rate of energy transfer between the lasers $\sim \kappa c / n$.

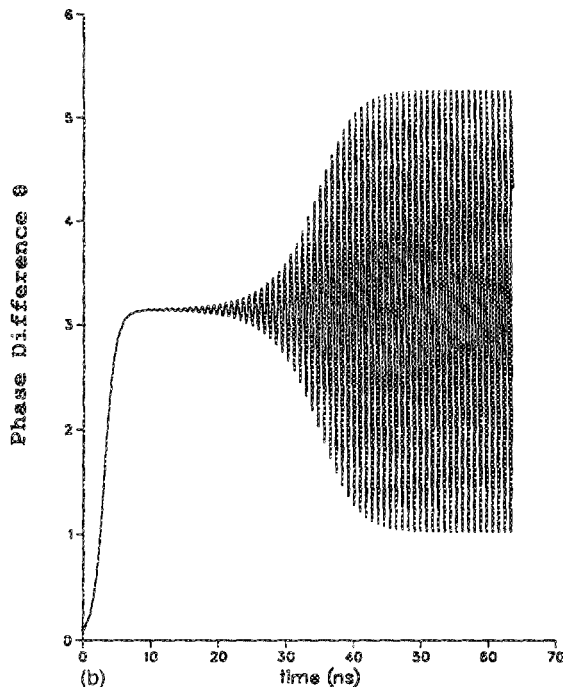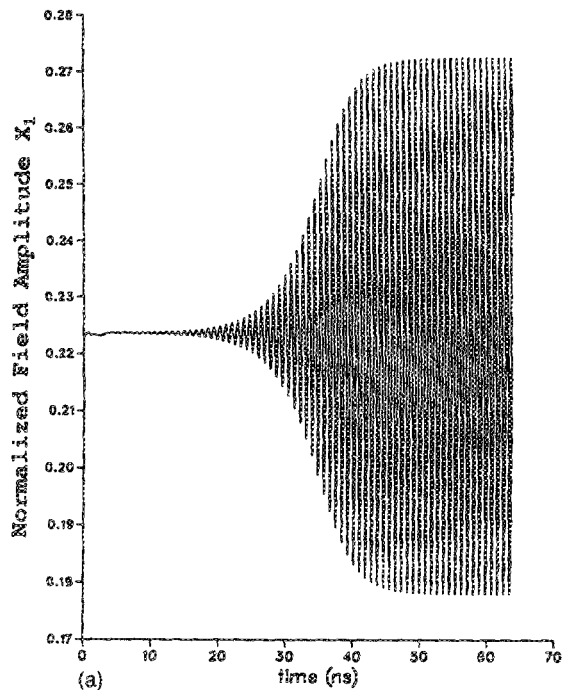The origin of the instability is the amplitude phase cou-

(a)

(b)

FIG. 2. Periodic oscillations of (a) field amplitude and (b) relative phase in the unstable domain. Here $p = 0.05$ and $\eta = 10^{-4}$. For larger values of η, the amplitude oscillations are aperiodic and the phase evolution is unbounded.

pling that occurs in semiconductor lasers and is described by the linewidth enhancement factor α. Note that the instability disappears as $\alpha \to 0$. Also, the stability domain increases as $T \to 0$. An amplitude fluctuation in one laser leads to a carrier density fluctuation and (through α) a phase fluctuation in that same laser. A change in relative phase leads to an amplitude change in the second laser and an accompanying change in its carrier density. Sustained oscillations can be expected when the lifetime of the carriers (τ_s) is longer than or comparable to the coupling time ($\tau_\kappa = n/\kappa c$).

The stability analysis presented here has shown that phase locking between adjacent laser elements can be destabilized by amplitude phase coupling. A similar result has been obtained in the problem of semiconductor laser injection locking.[8] It is not yet clear how the presence of more than two lasers will affect the stability of the array. Numerical solutions of the array equations show that the instability persists for particular parameter values. A general stability analysis for large N is in progress. Also not included in the present analysis is the effect of damping factors such as nonlinear gain and spontaneous emission into the lasing mode. These effects may help reduce the instability domain somewhat.

In conclusion, we have shown that amplitude phase coupling can lead to a self-pulsing instability in coherently coupled semiconductor lasers.

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