

ratios are plotted in Fig. 4 for each of the  $\gamma$ 's given above.  $T_0$  was determined using a thermal velocity of  $1.4 \times 10^4$  cm/sec. Shown also in Fig. 4 is a plot (solid line) of the ratio as predicted by the theory. For this case,  $\theta \sim 2.0^\circ$ . Note that as  $\gamma \rightarrow \infty$  the ratio is asymptotic to unity indicating no motion effects.

It is interesting to note that two-photon processes provide an additional approach to DFWM with applications to phase conjugation<sup>11</sup> and high-resolution spectroscopy.<sup>12</sup> A temporal grating is produced by the action of the two counter-propagating pump waves. The Doppler-free nature of the interaction as observed by Liao and co-workers eliminates<sup>13,14</sup> the effects of thermal motion washout and optimizes the interaction by exciting equally all atoms in the sample. Furthermore, formation of the temporal grating requires that the two linearly polarized pump waves have a nonzero scalar product. The phase conjugate signal is always polarized in the same direction as the probe wave. Again, one can write out the macroscopic polarization density as

$$\mathbf{P}_2 = C(\mathbf{E}_f \cdot \mathbf{E}_b)\mathbf{E}_p^*, \quad (5)$$

where the coefficient  $C$  contains all the dynamical parameters. In the case of nonresonant isotropic media such as  $\text{CS}_2$ ,<sup>15</sup> the macroscopic polarization is just given by

$$\mathbf{P} = A[\mathbf{E}_f \cdot \mathbf{E}_p^*]\mathbf{E}_b + (\mathbf{E}_b \cdot \mathbf{E}_p^*)\mathbf{E}_f + C(\mathbf{E}_f \cdot \mathbf{E}_b)\mathbf{E}_p^*.$$

We are currently experimentally investigating these effects in several atomic and molecular systems.

In conclusion we have demonstrated the ability of DFWM to provide an output signal whose polarization is different from the probe. Furthermore, using this discrimi-

nation technique we demonstrated in quantitative agreement with theory the washout due to motion. These effects are important from the standpoint of overall efficiency.

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## Theory of bistability in nonlinear distributed feedback structures

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We show that optical bistability can occur in a distributed feedback structure with an intensity-dependent refractive index. Analytical expressions for the transmissivity are obtained and a comparison with Fabry-Perot-type devices is presented.

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It has been established theoretically<sup>1</sup> and experimentally<sup>2</sup> that a Fabry-Perot interferometer filled with a nonlinear index medium exhibits hysteresis and bistability in its response to optical inputs. Such a device has potential applications as an optical transistor, pulse shaper, memory element, and differential amplifier. The underlying physical requirements are an intensity-dependent refractive index and an optical feedback mechanism. Previous theory and experiments on optical bistability have employed systems with lumped feedback; i.e., the feedback mechanism is spatially localized at the ends of a homogenous nonlinear medium either as mirrors<sup>2</sup> or Bragg reflectors.<sup>3</sup> In this letter we present an exactly solvable model of bistability in a device in

which the feedback mechanism is distributed throughout the nonlinear medium as a periodic variation of the linear refractive index.

The nonlinear distributed feedback structure (DFB) has a threshold for bistability which is comparable to the Fabry-Perot structure (FP) and has several distinct features. These include the possibility of a truly bistable device (FP devices are all multistable), absence of the strong optical limiting seen in FP's (except for DFB's with very large coupling coefficients), a decrease in optical hysteresis width with increasing input intensity, and the characteristic monochromatic spectral response of DFB structures. In addition to its obvious compatibility with integrated optics, the DFB

bistable device offers the advantage of control over the input/output characteristics by tailoring the DFB transmission through tapering the coupling strength or chirping the frequency of its grating.

The simplest approximate theories of lumped feedback devices<sup>1</sup> employ a "mean-field" approach which treats the intracavity field as independent of  $z$ , the distance in the propagation direction. This approach cannot be justified in a DFB structure where the fields are strongly  $z$  dependent. We, therefore, use a more exact coupled-wave theory which takes propagation effects into account. Consider a lossless isotropic medium of length  $L$  whose linear refractive index is

$$n(z) = n_0 + n_1 \cos(2\beta_0 z + \Phi), \quad (1)$$

where  $n_1 \ll n_0$  and  $\Phi$  is a constant phase. In addition, include an intensity-dependent refractive index term, so that the polarization density due to a single monochromatic beam of field strength  $E$  is

$$P = [n^2(z) - 1]E/4\pi + n_0 n_2 |E|^2 E/4\pi, \quad (2)$$

where  $n_2$  is the nonlinear index. Near the Bragg frequency one can write the field in the medium as a sum of forward and backward waves:

$$E(z) = E_F(z) \exp(i\beta z) + E_B(z) \exp(-i\beta z), \quad (3)$$

where  $\beta = n_0 \omega/c$ . Using Eq. (2) in Maxwell's wave equation we obtain, in the slowly varying envelope approximation,

$$-iE'_F = \kappa E_B \exp[-i(2\Delta\beta z - \Phi)] + \alpha [ |E_F|^2 + 2|E_B|^2 ] E_F, \quad (4a)$$

$$iE'_B = \kappa E_F \exp[i(2\Delta\beta z - \Phi)] + \alpha [ 2|E_F|^2 + |E_B|^2 ] E_B. \quad (4b)$$

Here the primes denote  $d/dz$ ,  $\alpha = n_2 \omega/2c$ ,  $\Delta\beta = \beta - \beta_0$ , and, in the notation of Kogelnik and Shank,<sup>4</sup> we define a coupling constant  $\kappa \equiv \frac{1}{2} \beta_0 n_1/n_0$ .

Equations (4) can be solved by the methods of Armstrong *et al.*<sup>5</sup> for harmonic generation or by the more general Lagrangian formulation of Marburger and Lam.<sup>6</sup> Following Ref. 5, we write  $E_F = |E_F| \exp(i\phi_F)$  and  $E_B = |E_B| \exp(i\phi_B)$  and integrate the real and imaginary parts of Eq. (4). This procedure leads to two conserved quantities

$$|E_T|^2 = |E_F|^2 - |E_B|^2$$

and

$$\Gamma = |E_F| |E_B| \cos \psi + (2\Delta\beta + 3\alpha |E_B|^2) |E_F|^2 (2\kappa)^{-1},$$

where  $|E_T|^2$  is the transmitted flux and  $\psi = 2\Delta\beta z + \phi_F - \phi_B + \Phi$ . These constants are then used to obtain an equation for the forward flux within the structure:

$$\left(\frac{1}{2} Ly'\right)^2 = (y - J)[(\kappa L)^2 y - (y - J)(\Delta\beta L + 2y)^2] \equiv Q(y), \quad (5)$$

with  $y = |E_F|^2/|E_C|^2$ ,  $J = |E_T|^2/|E_C|^2$ , and a "critical intensity"  $|E_C|^2 = 4n_0 \lambda / 3\pi n_2 L$ . At the input of the structure, we have  $y = I$ , the input intensity normalized to  $|E_C|^2$ . With this boundary condition Eq. (5) can be integrated to yield a relation between incident and transmitted intensities in terms of elliptic functions. The result is<sup>7</sup>

$$I = y_3(J) + \frac{y_3(J) - y_2(J)}{[y_3(J) - y_2(J)][y_1(J) - y_3(J)]^{-1} \text{sn}^2(u|m) - 1}, \quad (6)$$

where  $y_1 > y_2 > y_3 > y_4$  are the roots of  $Q(y) = 0$ , and are, in general, functions of  $J$ . The function  $\text{sn}(u|m)$  is a Jacobian elliptic function with argument

$$u = 2[(y_1 - y_3)(y_2 - y_4)]^{1/2} \quad (7)$$

and modulus

$$m = \frac{(y_1 - y_2)(y_3 - y_4)}{(y_1 - y_3)(y_2 - y_4)}. \quad (8)$$

At the Bragg frequency,  $\Delta\beta L = 0$ , and the roots of  $Q(y)$  are  $y_1 = \frac{1}{2}\{J + [J^2 + (\kappa L)^2]^{1/2}\}$ ,  $y_2 = J$ ,  $y_3 = 0$ , and  $y_4 = \frac{1}{2}\{J - [J^2 + (\kappa L)^2]^{1/2}\}$ .

From Eq. (6) we obtain for the on-resonance transmissivity of the structure,

$$T = J/I = 2(1 + nd \{ 2[(\kappa L)^2 + J^2]^{1/2} [1 + (J/\kappa L)^2]^{-1} \})^{-1}. \quad (9)$$

Here  $nd(u|m)$  is another Jacobian elliptic function and the dimensionless fields are related to esu field strengths by

$$I, J = \frac{3\pi L}{4\lambda} \frac{n_2}{n_0} |E|^2,$$

We remark that the geometrical construction introduced in Ref. 1 can be applied to this problem if the linear transmission function of a Bragg filter is replaced by the nonlinear relation of Eq. (9). Note that when the nonlinear refractive index vanishes, the modulus of  $nd(u|m)$  becomes unity and we recover the usual expression for the on-resonance transmission of a Bragg filter:

$$T = J/I = \text{sech}^2(\kappa L).$$

In Fig. 1, we plot Eq. (6) as a relation between dimensionless input and output intensities for several values of the detuning parameter  $\Delta\beta L$  and a coupling constant  $\kappa L = 2$ . As in all bistable devices, the negative slope regions are unstable; hence, the output shows discontinuous jumps at certain critical input intensities. As the input is slowly varied, a hysteresis loop is traced out whose width depends on  $\Delta\beta L$

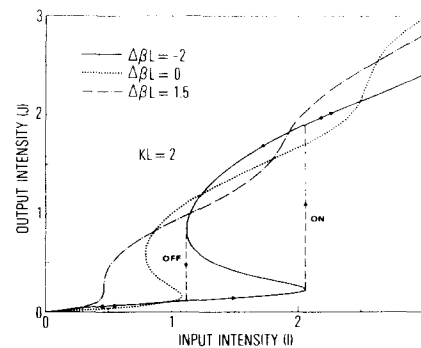


FIG. 1. Output intensity versus input intensity for a DFB structure with  $\kappa L = 2$  and three different values of detuning.

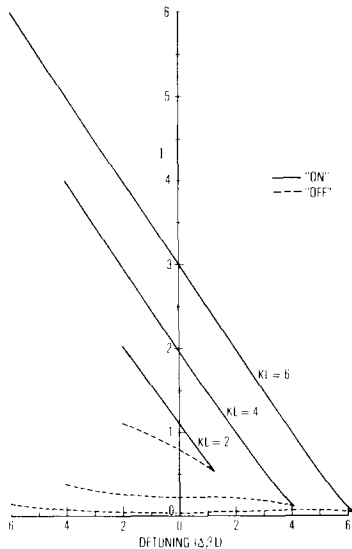


FIG. 2. "On" and "off" fields for bistable operation versus detuning for  $\kappa L = 2, 4,$  and  $6$ .

and  $\kappa L$ . Figure 2 summarizes the dependence of the width of the hysteresis zone on the detuning and coupling constant. It is clear from this plot that to minimize the switching fields one needs to operate on the high-frequency side of the Bragg resonance with a high coupling constant. The dependence of the properties of the nonlinear DFB on the coupling constant  $\kappa L$  is shown in Fig. 3, obtained from Eq. (9). For low values of  $\kappa L$  the feedback is insufficient to create bistability. Figure 3(a) shows the optical transistor mode of operation which occurs at  $\kappa L \simeq 1.4$ . A single bistable regime occurs at  $\kappa L \simeq 2$  [Fig. 3(b)], and larger values of  $\kappa L$  lead to multiple-switching phenomena [Fig. 3(c)]. For comparison, in Fig. 3(d), we plot the transmission of a nonlinear Fabry-Perot whose maximum reflectivity is about equal to that of a DFB with  $\kappa L = 4$ .

The origin of the striking difference between the DFB and FP can be seen from their very different linear transmission characteristics. In the FP, 100% transmission occurs over a very narrow but periodic frequency range, which results in multistable behavior with strong optical limiting. In the DFB, with relatively low values of  $\kappa L$ , the transmission is high everywhere except near the Bragg frequency, so that only a single bistable region is obtained, with no optical limiting. At large values of  $\kappa L$ , the DFB transmission function begins to have strong periodic sidelobes, which means that multistability, as well as optical limiting begins to occur.

The DFB transmission function can be tailored by varying the grating frequency ( $\beta_0$ ) or its depth ( $n_1$ ) to increase or decrease the sidelobes or to broaden the response around

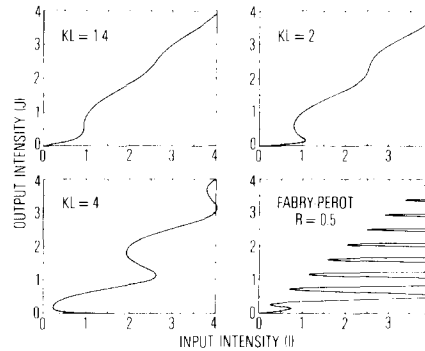


FIG. 3. Output intensity versus output intensity (a)  $\kappa L = 1.4$ , optical transistor action; (b)  $\kappa L = 2$ , single bistable region; (c)  $\kappa L = 4$ , multiple switching; (d) a comparison Fabry-Perot with mirror reflectivity  $R = 0.5$ . Note the strong power-limiting effect.

the Bragg frequency. This makes it possible in principle to modify the input/output characteristics of the DFB bistable optical device (BOD) at will. Finally, since the reflectivity of the DFB BOD is high on resonance and low elsewhere, it is similar to the transmission mode of a FP, and thus the reflective mode of the DFB BOD should show strong optical limiting. In fact, by tailoring the grating to eliminate sidelobes, the DFB BOD used in the reflection mode should be a better optical limiter than the FP.

To estimate the required fields for bistable operation, a lossless GaAs Bragg reflector with  $\kappa L = 6$ ,  $n_2 = 1.6 \times 10^{-10}$  esu,  $\lambda = 1.06 \mu\text{m}$ , and  $L = 1$  cm has a threshold switching intensity of order  $85 \text{ MW/cm}^2$ . This switching intensity is comparable to the  $75 \text{ MW/cm}^2$  required for a Fabry-Perot of equal length and mirrors of reflectivity 0.9.

In conclusion, we have demonstrated that a distributed feedback structure with a nonlinear refractive index is bistable and have obtained analytical expressions for its transmissivity. The physical mechanism for the bistability is the optically induced changes in the Bragg condition for coupling between forward and backward waves within the structure.

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