OPTICAL BISTABILITY IN PERIODIC STRUCTURES AND IN FOUR-WAVE MIXING PROCESSES

by

Herbert Graves Winful

A Dissertation Presented to the

FACULTY OF THE GRADUATE SCHOOL

UNIVERSITY OF SOUTHERN CALIFORNIA

In Partial Fulfillment of the

Requirements for the Degree

DOCTOR OF PHILOSOPHY

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INTRODUCTION

Optical bistability is a phenomenon in which an optical system displays two stable output intensity states for the same input intensity. The two states are linked by a hysteresis loop, and transitions between them can be obtained by varying the input intensity adiabatically (Fig. (1.1)). There are many possible applications of this phenomenon in the areas of optical communications and optical signal processing. Among these applications are binary logic operations, differential amplification, pulse shaping and regeneration, optical limiting and ultrafast switching [1]. Quite apart from these applications, optical bistability is of great interest from a fundamental physics point of view. It is an example of a general class of nonlinear systems maintained under conditions far from equilibrium by an external driving input. Such systems are known to exhibit phase transitions and hysteresis phenomena [2].

The physical requirements for bistability are i) a medium whose refractive index or absorption is intensity-dependent, and ii) some means of feedback such that the medium's transmission depends on the output intensity. While the original proposal for a bistable optical device

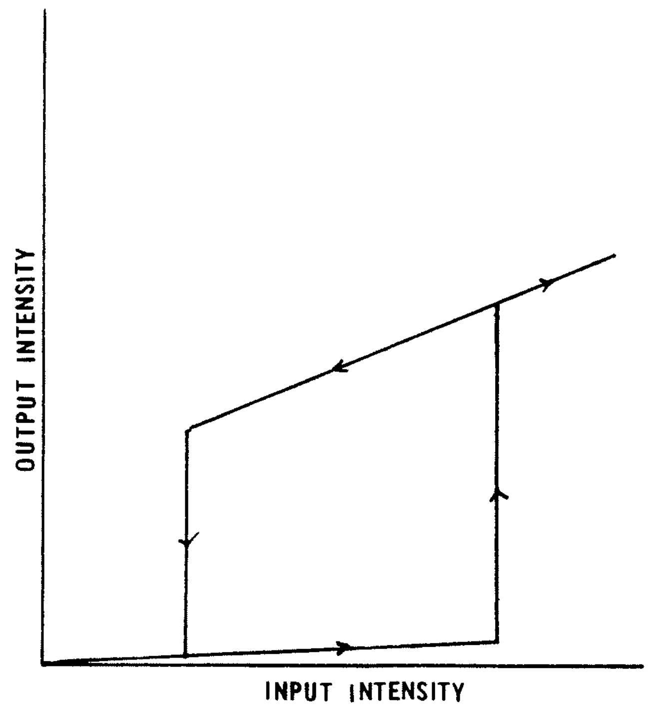

Fig. (1.1): Transmission characteristic of a typical bistable optical device.

employed a saturable absorber as the nonlinear medium [1], most of the demonstrated devices operate via the intensity-dependent refractive index, with feedback provided by Fabry-Perot mirrors [3]. It is also possible to create the nonlinearity and feedback mechanism electronically by detecting the output light of an electro-optic modulator and feeding it back as a voltage onto the modulator [4]. Such hybrid electrical/optical devices are of interest because they require low driving intensities and can be used to model the behaviour of nonlinear Fabry-Perots in a convenient manner [5].

Both the Fabry-Perot and the hybrid device are examples of lumped feedback systems in which the feedback mechanism is spatially localized at the ends of a homogeneous nonlinear medium. This dissertation considers for the first time in bistability theory the concept of a feedback mechanism distributed throughout and integrated within the nonlinear medium. Two cases are considered. In the first, feedback is provided by Bragg scattering from a fixed periodic perturbation in the boundary or the refractive index of a nonlinear thin film. In the second, there is an internal self-induced feedback resulting from the instantaneous creation of gratings within the nonlinear medium by the interacting waves themselves. This occurs, for example, in the degenerate four-wave mixing process which results in phase-conjugate signals [6]. In both cases exact analytic solutions are obtained for the nonlinear coupled-wave equations in terms of Jacobi elliptic functions. Based on these solutions some new and interesting predictions are made concerning bistability in periodic structures and in degenerate four-wave mixing.

The organization of this dissertation is as follows. In Chapter II we begin with a discussion of some current trends in the field of optical bistability with emphasis on experimental results. This is followed by a review of some of the theoretical models of bistability.

In Chapter III we solve the problem of optical bistability in a periodic structure. The effects of loss, chirp and taper are considered, as well as the effect of a saturable nonlinear index. Design considerations for a practical bistable device with distributed feedback are presented.

In Chapter IV we review some aspects of the general interaction between four waves in a nonlinear medium. We then derive exact solutions for a model of degenerate four-wave mixing that includes pump depletion and intensity-dependent phase shifts. The fields are shown to exhibit hysteresis and bistability. We conclude with suggestions for a possible experiment.

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CHAPTER II

THE THEORY AND PRACTICE OF OPTICAL BISTABILITY

A. Introduction

Current research in optical bistability is proceeding along two very distinct fronts. On the one hand there is the applied research that is concerned with developing the ultimate bistable optical device. This work has led to the discovery of some unusually large third order nonlinear optical coefficients in semiconductors and has spurred theoretical efforts to understand their origins. On the other hand there is a large body of theoretical work that is concerned with models of homogeneously broadened two-level atoms in resonant cavities driven by intense coherent radiation. Such research has led to some interesting predictions concerning the spectra of the transmitted and fluorescent radiation, the critical behavior, instabilities and hysteresis phenomena that may occur in such systems. It is the purpose of this chapter to review some of the theoretical models of bistability that are in current use and to indicate the thrust of the applied research.

B. Applied Research

We begin with a brief discussion of the nonlinear Fabry-Perot devices that operate via an intensity-dependent refractive index. To see how such devices operate, suppose that at low input intensity the Fabry-Perot is detuned from the incident light wavelength so that its transmission is low, and little light enters the cavity. As the input intensity is increased more light enters the cavity and changes the refractive index of the non-linear medium thus tuning the cavity closer to a resonance. This increases the transmission of the cavity allowing more light to enter and change the index even more. Eventually the cavity shifts into resonance and the device turns on to a high transmission state. As the input is reduced the high Q of the cavity enables it to retain a sufficiently large internal field that the high transmission state is maintained. The input has to drop below the original switch-up intensity before the transmission falls suddenly to a low value. This results in the hysteretic response curve shown in Fig. (1.1).

One of the main goals of the applied research in bistability is the development of a fast (picosecond), solid state, low-power (milliwatt) bistable optical device that operates at room temperature. The requirements of speed and low-power have led to a search for materials with large electronic nonlinearities. One approach to this problem is to use the resonant enhancement of the third order susceptibility that occurs near the bandgap of semiconductors. The current leader in this derby is the semiconductor InSb which has been found to have a $\chi^{(3)}$ on the order of 1 e.s.u. at 77°K[1].

The microscopic origin of this enormous nonlinearity is a subject of current study, but three possible mechanisms have been proposed: i) saturation of a set of two-level oscillators associated with conduction-valence band transitions; ii) the shift of the absorption edge due to band filling; and iii) the creation of an electron-hole plasma. The weak absorption in the band tail below the fundamental absorption edge is responsible for the excitation that gives rise to all three listed mechanisms. For the purpose of a bistable optical device it is important that the transitions involved be weakly absorbing since too much loss will ruin the cavity finesse and destroy bistability. We remark that room temperature operation of the InSb device has not been attempted since the band gap shifts outside the wavelength range of the CO laser used in these studies. Up to 77°K, bistability has been observed in InSb with only $15\mu W/(\mu m)^2$ holding intensity.

Another material that is currently receiving a great deal of attention is GaAs. Bistability has been observed in GaAs at temperatures of up to 120°K with a holding intensity of $1mW/(\mu m)^2$. The mechanism involved here is the saturation of the free exciton resonance [2]. Clearly room temperature operation using this mechanism is unlikely since the exciton feature disappears at temperatures higher than about 200°K. The response time of the GaAs device is on the order of 1ns for turn-on, and 40ns for turn-off.

A radical approach to the problem of finding solid state materials with large nonlinearities at room temperature is to create them artificially. The technology of molecular beam epitaxy (MBE) makes it possible to grow single layers of atoms on a substrate. By growing, for example, alternate layers of GaAs and GaAlAs in a periodic manner, it is possible to create a "superlattice" whose nonlinear optical properties may be quite different from those of either material [3].

The ultimate speed of nonlinear Fabry-Perot devices is limited by the material response time and the cavity round trip time. To circumvent the cavity lifetime limitation, it is possible to use the hysteresis and bistability that occurs on reflection from a single interface between a linear and a nonlinear medium [4]. This phenomenon is due to an intensity tuning of the refractive index of the nonlinear medium so that conditions for total internal reflection are either satisfied or not, depending on the history of the excitation. Devices based on this phenomenon may be capable of extremely rapid switching action since no cavities are involved.

C. Theoretical Approaches

A large body of literature now exists that treats both the macroscopic and microscopic properties of optically bistable systems. In this section we review some of the better known theories of optical bistability.

1. The Model of Szöke et al.

Szöke, Daneu, Goldhar, and Kurnit in the first published work on optical bistability [5] considered a Fabry-Perot resonator filled with a saturable absorber. The absorber is modelled as a simple two-level system obeying the rate equation

$$\frac{dn_u}{dt} = \frac{\alpha I_2}{\hbar\omega} - \frac{n_u}{\tau_R} , \qquad (2.1)$$

where n_u is the population of the upper level, τ_R is the relaxation time of that state, α is the intensity-dependent absorption coefficient and $I_2/\hbar\omega$ is the incident photon flux. Standard theory [6] gives the absorption coefficient as

$$\alpha/\alpha_0 = (n_1 - n_u)/N , \qquad (2.2)$$

where α_0 is the unsaturated absorption coefficient, N is the density of atoms and $n_1 = N - n_u$ is the population of the lower level. The steady state solution for n_u obtained from (2.1) is then substituted into Eq. (2.2) to yield

$$\alpha = \frac{\alpha_0}{1 + I_2/I_0} ,$$

where $I_0 = \frac{\hbar\omega}{2\tau_R} \frac{N}{\alpha_0}$.

Having modelled the saturable absorber it is then necessary to determine the relations between field amplitudes at the input and output of the Fabry-Perot. Fig. (2.1) defines the field quantities of interest. At the input, the following relations hold:

$$E_B(0) = E_F(0)e^{-(i2kl + \alpha l)} ,$$

$$E_F(0) = \tau E_I + \rho E_B(0) , \qquad (2.4)$$

where ρ and τ are the amplitude reflection and transmission coefficients respectively. Equations (2.4) can be solved to give

$$E_F(0) = \frac{\tau E_I}{(1 - \rho^2 e^{-i2kl} e^{\alpha l})} . \qquad (2.5)$$

The transmitted field is then given by

$$E_T = \tau E_F(0) e^{-ikl} e^{-\alpha l/2} . \qquad (2.6)$$

To make further analytical progress, Szöke et al. now assume high reflectivity mirrors such that $T = 1 - R << 1$, and weak absorption so that $\alpha l << 1$. Then defining a dimensionless parameter $K = R\alpha l/(1-R)$, the internal field on

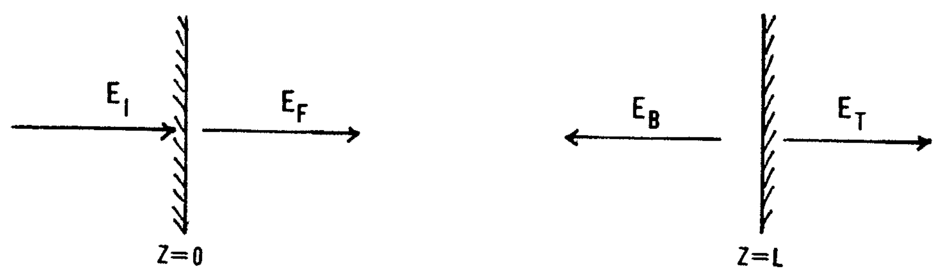

Fig. (2.1): Schematic of a Fabry-Perot interferometer filled with a saturable absorber.

resonance is found from Eq. (2.5) to be

$$I_2 = |E_F(0)|^2 = \frac{I_1}{T} \frac{1}{(1 + K)^2} , \tag{2.7}$$

where $I_1 = |E_I|^2$ and $K >> 1$. Substitution of Eq. (2.3) into (2.7) yields a relation between incident and cavity fields of the form

$$z_1 = z_2 \left(1 + \frac{K_0}{1 + z_2} \right)^2 , \tag{2.8}$$

with $z_1 = I_1/I_0 T$, $z_2 = I_2/I_0$ and $K_0 = R\alpha_0 l/T$. The intensity in the cavity can be a multivalued function of the input depending on the size of the parameter K_0. Taking a derivative of (2.8) shows that the curve z_1 vs z_2 has a maximum and minimum if the condition

$$\frac{R\alpha l}{T} > 8 \tag{2.9}$$

is satisfied.

The theory of Szöke et al. thus gives a necessary condition for bistability in a saturable resonator. It is a mean field theory which ignores the standing wave nature of the cavity field.

2. McCall's Theory

McCall [7] also considered the behaviour of a saturable absorber in a Fabry-Perot cavity. (His somewhat more general theory takes into account the effects of standing waves and atomic coherence.) In his model the absorber is described by Bloch's equations with relaxation times T_1 and T_2' representing the decay of population and coherence respectively. These equations are

$$\frac{\partial v}{\partial t} = -w\kappa E_R - v/T_2' \tag{2.10a}$$

$$\frac{\partial u}{\partial t} = w\kappa E_I - u/T_2' \tag{2.10b}$$

$$\frac{\partial w}{\partial t} = v\kappa E_R - u\kappa E_I - (w + 1)/T_1. \tag{2.10c}$$

Here u and v are the in-phase and quadrature components of the polarization $Q = u + iv$ induced by the electric field $E = E_R + iE_I$. w is proportional to the population difference and $\kappa = 2p/\hbar$, p being the dipole moment and $\hbar$ being Planck's constant divided by 2π.

The steady state polarization obtained from Bloch's equations is then substituted into Maxwell's equations in the slowly-varying-envelope approximation:

$$\frac{\partial E}{\partial z} + \frac{n}{c}\frac{\partial E}{\partial t} = \frac{i2\pi\omega Np}{nc} Q \tag{2.11}$$

where N is the atomic density. McCall expresses the cavity field as

$$E(z,t) = E_F e^{-i(\omega t+kz)} + c.c. + E_B e^{-i(\omega t-kz)} + c.c.$$

where E_F and E_B are the slowly-varying amplitudes of the forward and backward propagating waves. Inclusion of the spatial variations of the intracavity fields makes the resultant equations analytically intractable. McCall thus resorts to a numerical integration of the SVEA equation and shows that the transmitted field is a multivalued function of the incident field.

3. Bonifacio-Lugiato

The contribution of Bonifacio and Lugiato was to ignore the complication of spatial field variations (Mean Field Approximation) and thus obtain simple analytic expressions for optical bistability based on the Maxwell-Bloch equations [8]. If one defines incident (y) and transmitted (x) field quantities through

$$y = \frac{p}{\hbar} \frac{E_I}{(T/T_1T_2')^{1/2}} \quad \text{and} \quad x = \frac{p}{\hbar} \frac{E_T}{(T/T_1T_2')^{1/2}}$$

where T is the mirror transmission, then it is easy to show from Eqs. (2.10) that in the steady state,

$$y = x + \frac{2Cx}{1 + x^2} . \qquad (2.12)$$

Here $C = \frac{\alpha L}{2T}$, and it is clear that if we write $y = \sqrt{z}$, $x = \sqrt{z_2}$ and $K_0 = 2C$, the Bonifacio-Lugiato result is identical to that of Szöke et al.

4. Marburger-Felber

The theory of Marburger and Felber is a macroscopic one that starts with a given susceptibility [9]. It is not concerned with the microscopic origins of this susceptibility. It also deals with an intensity dependent refractive index rather than saturable absorption. Since most of the observed bistable devices operate via an intensity-dependent refractive index, the Marburger-Felber theory has proven to be quite useful for making quantitative estimates of switching power and for analyzing experimental results [10].

Their simplest approximate theory consists of the standard Fabry-Perot transmission equation with an intensity-dependent phase shift:

$$T = \frac{1}{1 + F\sin^2 \delta/2} \tag{2.13}$$

where $\delta = \delta_0 + v\,E^2$, v is proportional to the nonlinear index, E is the cavity field and $F = 4R/(1-R)^2$.

Marburger and Felber have also solved the problem of the nonlinear Fabry-Perot exactly, without invoking the slowly varying approximation. It turns out that when a cubic polarization of the form

$$P = \eta|E|^2E$$

is used in Maxwell's equations, exact solutions can be found in terms of elliptic functions.

In the next chapter, we solve the problem of optical bistability in a nonlinear periodic structure. Our analysis is closest in spirit to that of Marburger and Felber.

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